Hybrid systems, Lecture 9: Timed Automata

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07.04.2015

What is *Timed Automata*

Timed automata are a subclass of hybrid automata
$TA = \left(Q, X, \text{Init}, f, D, E, G, R\right)$

- $Q = \left\{q_1, \ldots, q_m\right\}, \quad X = \mathbb{R}_+^n, \quad Init \subseteq Q \times \left(0, \ldots, 0\right)^T$
- $f(q, x) = (1, \ldots, 1)^T$
- $E \subseteq Q \times Q$
- $D(q), G(e)$ are rectangular sets. (finite boolean combinations of constraints of the form $x_i \bowtie a_i$ $(\bowtie \in \{<, \leq, =, \geq, >\})$, $a_i \in \mathbb{Z}^+$)
- $R(e, x) = \{x'\}$, where $x_i' = 0$ or $x_i' = x_i$ for all $1 \leq i \leq n$

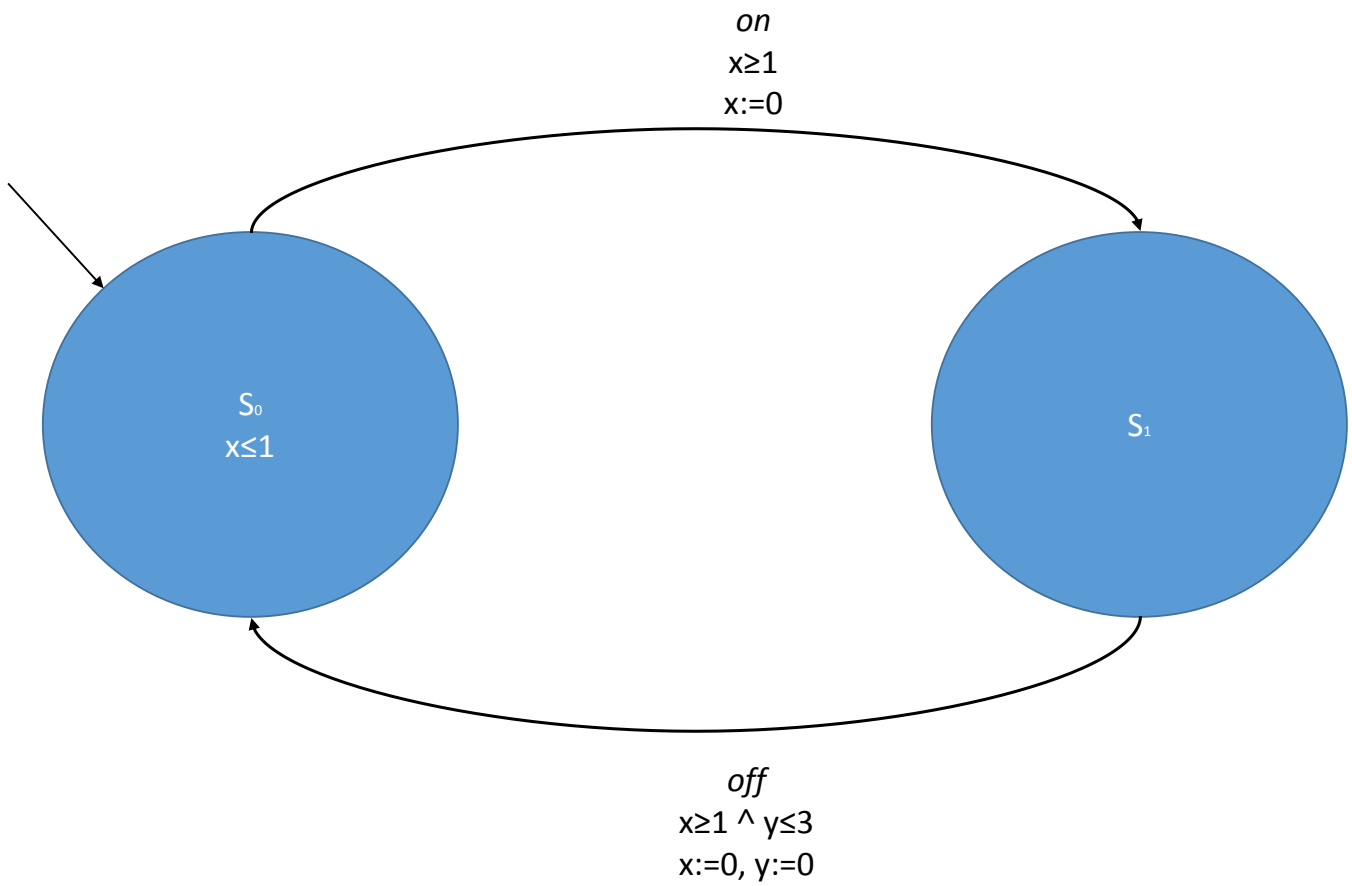
Labeled Timed Automata

Labeled timed automata are a subclass of hybrid automata

$TA = \left(Q, X, \text{Init}, \text{Act}, f, D, E, G, R\right)$

- $Q = \left\{q_1, \ldots, q_m\right\}, \quad X = \mathbb{R}_+^n, \quad \mathit{Init} \subseteq Q \times \left(0, \ldots, 0\right)^T$
- $f(q, x) = (1, \ldots, 1)^T$
- Act is the set of events
- $E \subseteq Q \times Q$
- $D(q), G(e)$ are rectangular sets. (finite boolean combinations of constraints of the form $x_i \bowtie a_i$ ($\bowtie \in \{<, \leq, =, \geq, >\}$), $a_i \in \mathbb{Z}^+$)
- $R(e, x) = \{x'\}$, where $x_i' = 0$ or $x_i' = x_i$ for all $1 \leq i \leq n$

Example



Timed Automata as Transition System

Let us interpret a timed automaton
$TA = \left(Q, X, \text{Init}, \text{Act}, f, D, E, G, R\right)$ as a transition system
$T_{TA} = \left(S\Sigma, \rightarrow, S_0 = \text{I}nit\right)$:

- $S = Q \times X$ and $(q, x) \in S$ denotes the state
- $\Sigma = \text{Act} \cup \mathbb{T}$, where the generators Act are the event names and $\mathbb{T}$ the continuous evolution
- $(q, x) \xrightarrow{\sigma} (q', x')$ for $+sigma \in \text{A}ct$ if
  - there exists $(q, \sigma, q') \in E$
  - x satisfies the guard $G(q, \sigma, q')$, $\{x'\} = R(x, (q, \sigma, q'))$, and x' satisfies the domain D(q').
- $(q, x) \xrightarrow{\mathbb{T}} (q, x')$ if $x' = x + \mathbb{T}$ and x' satisfies $D(q)$.

Topics for student presentations

- Equivalence and bisimulation of the transition system
- Languages of timed automata
- Reachability of timed automaton

Important information

NB! Test Nr. 1 Will take place on 14.04.2015

- You are allowed to use your own notes and handouts of lecture slides (should be printed).
- You are not allowed to use computers, books, phones and any other devices allowing communication.