Hybrid Systems, Lecture 3: Controls Systems (Reminder) Feedback

S. Nõmm

[1]Department of Computer Science, Tallinn University of Technology

17.02.2015

Feedback

Let us consider the system

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1}$$

Here and after this system will be referred as *system (1)* .
Without loss if generality assume that D is zero.

Our main goal is to choose control u such that it would define behaviour of the system (1)

- State feedback
- Output feedback

State feedback

Assume that the state of the system (1) is totally measurable A state feedback law has the following form

$$u = -Kx + k_r r$$

where r is the reference value for the output.
The closed loop dynamics of the system are given by

$$\dot{x} = (A - BK)x + Bk_r r$$

How to find K ?

How to find K

- Pole placement (eigenvalue assignment)
- Linear quadratic Regulators

Just some of the possible techniques

Pole placement (eigenvalue assignment)

If the feedback is given by

$$u = -Kx + k_r r$$

then the closed loop system

$$\dot{x} = (A - BK)x + Bk_r r$$

The gain K should be determined in a such form to guarantee the characteristic polynomial of the system

$$p(s) = s^n + p_1 s^{n-1} + \cdots + p_{n-1} s + p_n$$

Example 1

Linear Quadratic Regulator

A linear quadratic regulator minimizes the cost function

$$\tilde{J} = \int_0^{\infty} \left(x^T Q_z x + u^T Q_u u \right) dt$$

Here matrices Q_x and Q_u describe how much each state and input contribute to the overall cost.
Linear control law of the form

$$u = -Q_u^{-1} B^T P x$$

is the solution of the LQR problem. Where P is a positive definite, symmetric matrix that satisfies the *algebraic Riccati equation:*

$$PA + A^T P - PBQ_u^{-1} B^T P + Q_x = 0.$$

Example 2

Output feedback

Assume that the system is observable (the state may be estimated on the basis of known inputs and outputs). An observer (*dynamical system that estimates the state of another demonical system) is given by*

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$

by combining with state feedback control law one gets stabilizing controller

$$u = -K\hat{x} + Krr$$

Example 3.

Computer class practice

- Examples 1 -3 SciLab and xcos practice.
- Assignments given last week.
- Assignments for the next week.