Hybrid Systems
Lecture 1

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What is hybrid system?

- A hybrid system is a dynamical system with interacting time-triggered and event triggered dynamics
- For example differential equations and finite automata: $\dot{x} = f(x,u)$ and $q^{+} = g(q,v)$

State 1

State 2
Dynamics explaining behavior of this aircraft differ much from the one on the left side.

Course organization

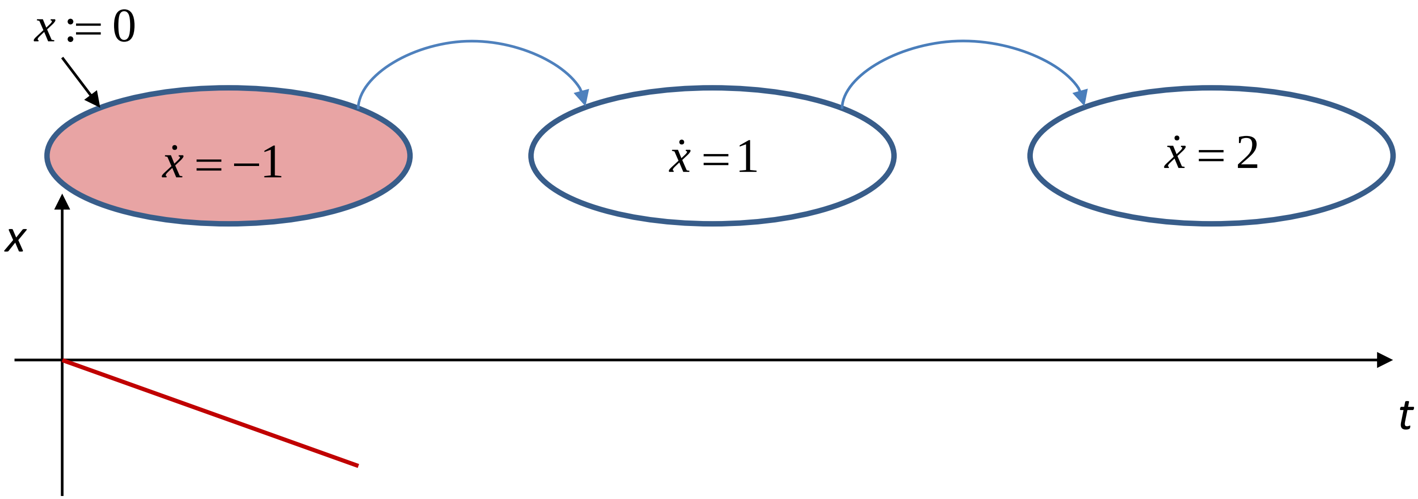
- **Contact: E-mail for Questions and Home assignments** **sven.nomm@gmail.com**
- **You may download the slides: TBA**
- **References:**
  - **Handbook of Hybrid Systems Control, Cambridge University Press, 2009, Editors:** JAN LUNZE & FRANÇOISE LAMNABHI-LAGARRIGUE
  - **Additionally some materials will be cited during the course and made available via webpage if necessary**

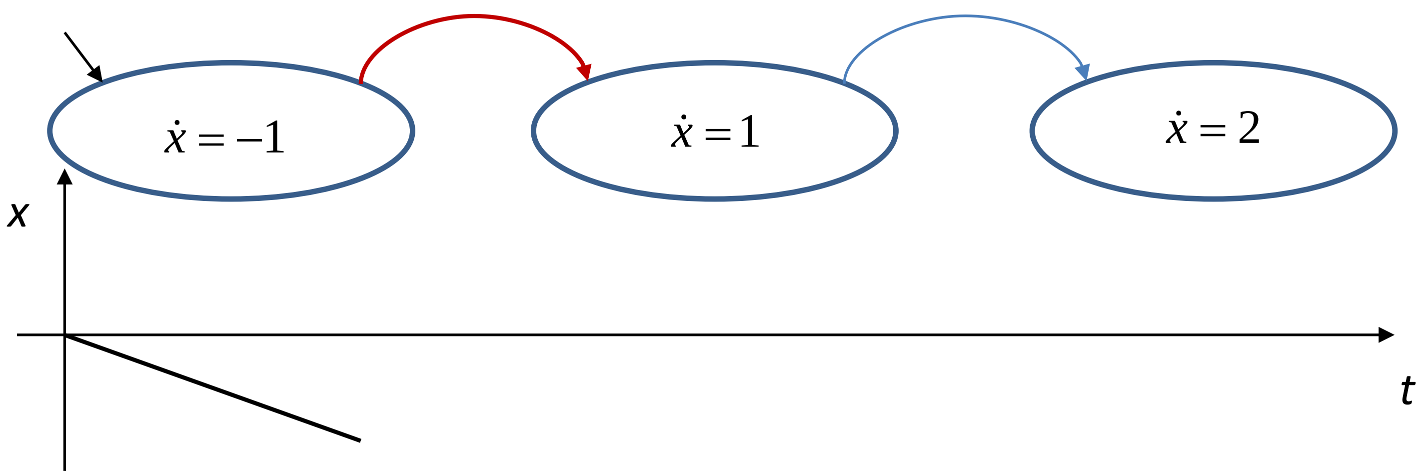
- The course consists of a) Theoretical lectures, Student presentations, Practical exercises in SciLab environment. The class is reserved on Tuesdays 16:00-17:30. Some times we will explore some examples together, sometimes I just be around to help you with your studies.
- Grading: Your final grade will be computed on the basis of the following tests:
  - two tests, each gives 10 % of final grade
  - two home assignments (followed by presentation), each gives 10 % of final grade
  - final project, gives 60% of the final grade

NB!!! In order to pass the course successfully you should complete all the assignments!!!

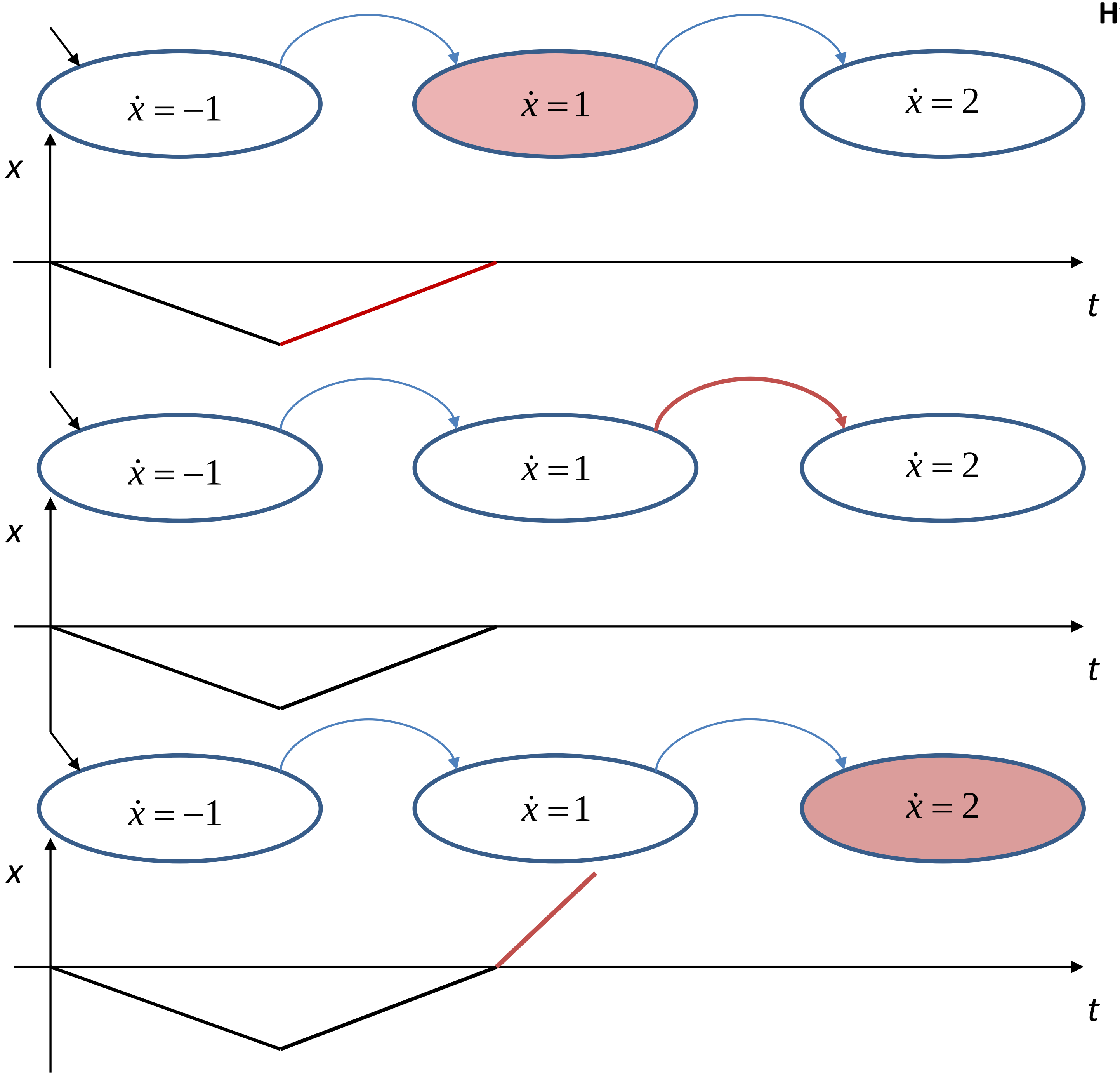
Simple example of a hybrid system

Let us suppose that one have to switch between 3 following systems with continuous dynamics $\dot{x} = -1; \quad \dot{x} = 1; \quad \dot{x} = 2;$





What is missing on this diagram?



Based on the example 1 (given by the lecturer) implement this system

Hybrid Automaton

- A hybrid automaton is a formal model of a hybrid system.
- A hybrid automaton is a transition system that is extended with continuous dynamics. It consists of locations, transitions, invariants, guards, *n-dimensional continuous* functions, jump functions, and synchronization labels.

- Formal definition of the hybrid automaton:
  - *A hybrid automaton H is a tuple* $H = (Q, V, f, Init, Inv, \Theta, G, R, \Sigma, \lambda)$,
    - $Q = \{q_1, \ldots, q_k\}$ *is a finite set of discrete states (control locations);*
    - $V = \{x_1, \ldots, x_n\}$ *is a finite set of continuous variables;*
    - $f : Q \times R^n \rightarrow R^n$ *is an activity function;*
    - $Init \subset Q \times R^n$ *is the set of initial states;*
    - $Inv : Q \rightarrow 2R^n$ *describe the invariants of the locations;*
    - $\Theta \subseteq Q \times Q$ *is the transition relation;*
    - $G : \Theta \rightarrow 2^{Rn}$ *is the guard condition;*
    - $R : \Theta \rightarrow 2^{Rn} \times 2^{Rn}$ *is the reset map;*
    - Σ *is a finite set of synchronization labels;*
    - $\lambda : \Theta \rightarrow \Sigma$ *is the labeling function.*

The automaton H describes a set of (hybrid) states $(q, \mathbf{x}) \in \mathbf{H} = \mathbf{Q} \times \mathbf{R}^n$.

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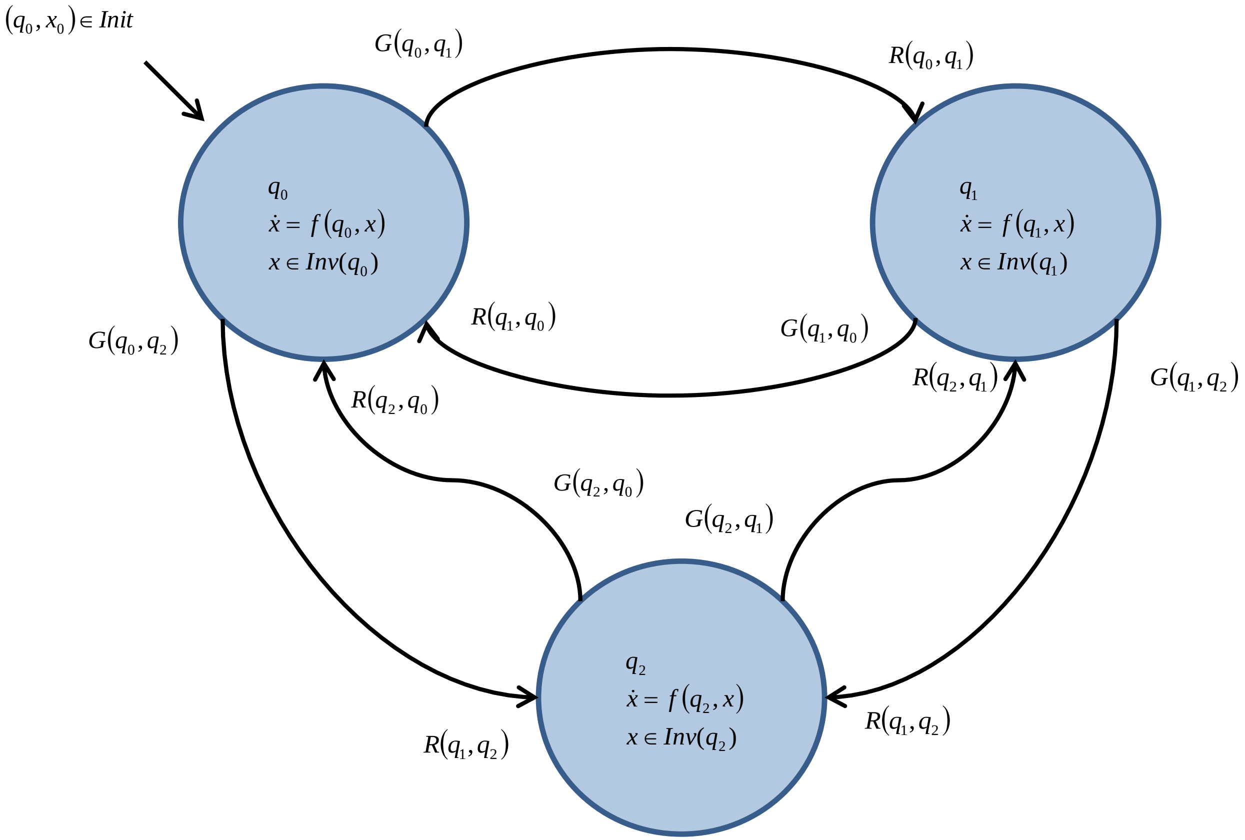
Specifies discrete dynamics

Describes continuous dynamics & its limitations

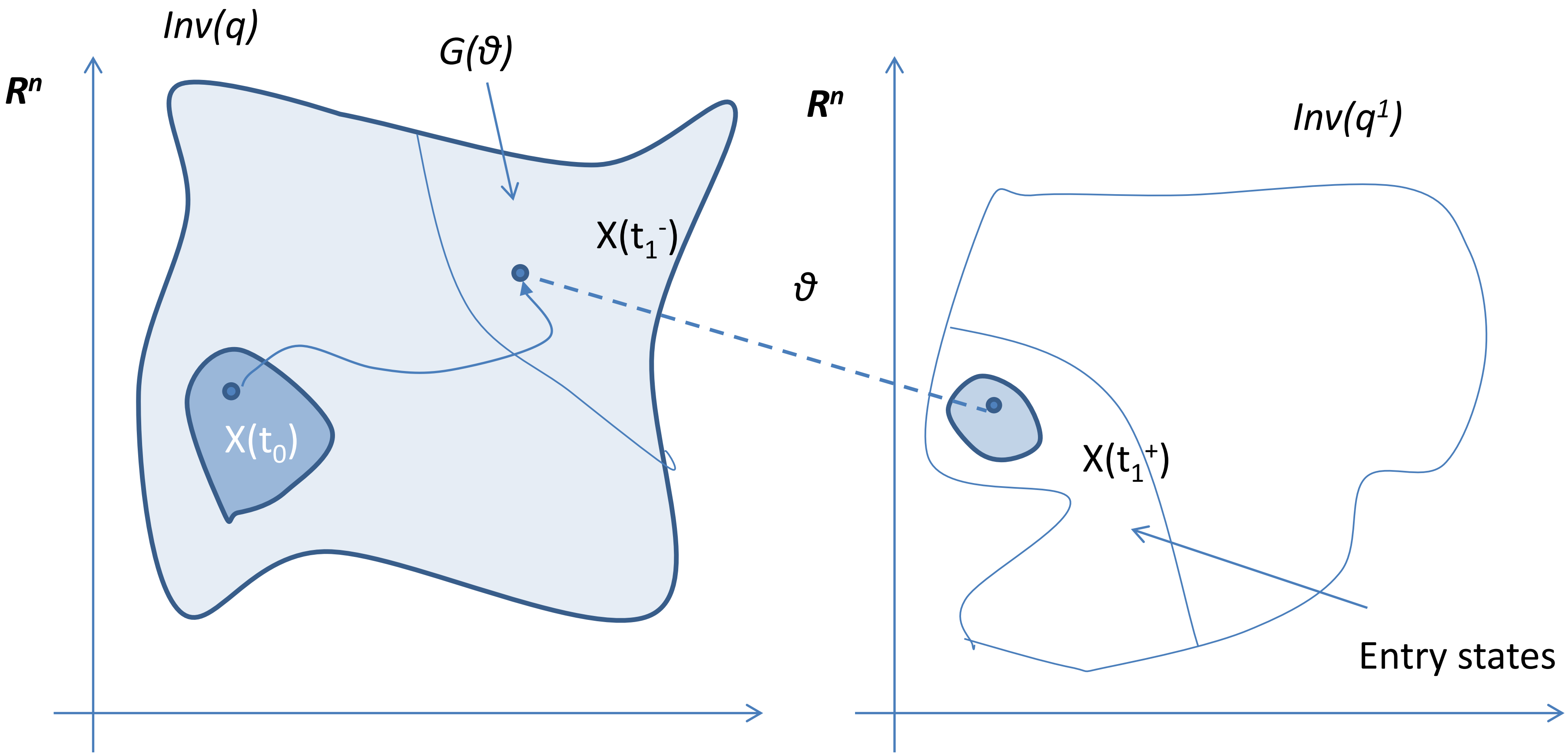
Necessary to synchronize different systems

Schematic representation of a hybrid automaton with three discrete states.

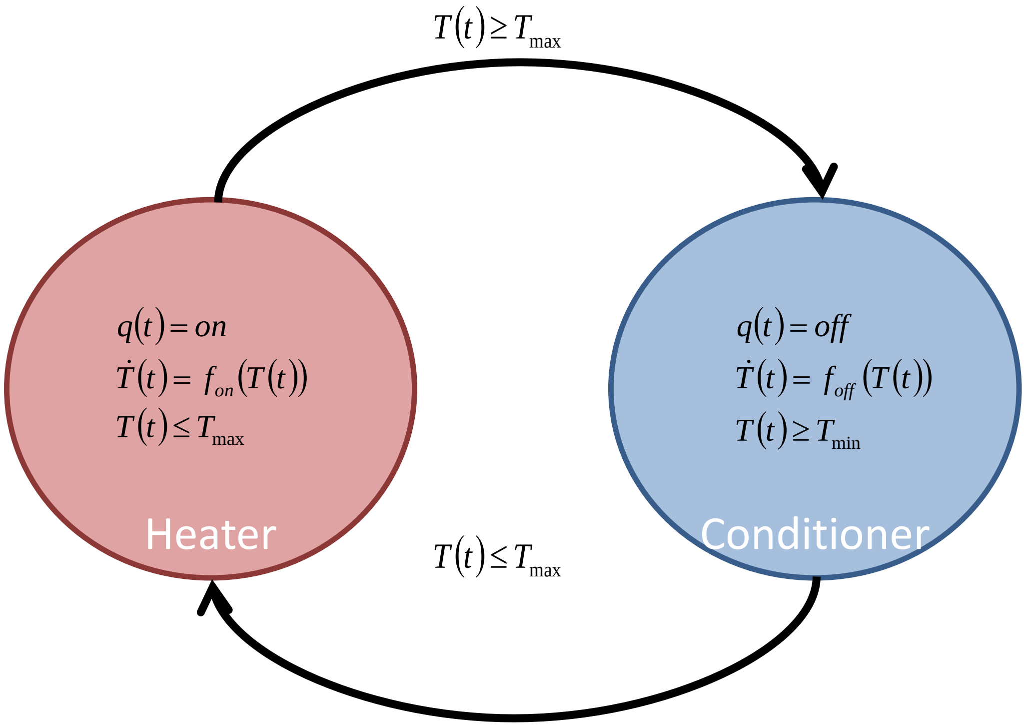


a finite set of initial states $Init \subseteq H$
an *invariant* mapping *Inv* :*Q*→R*n*;
a *guard* mapping *G* : *Θ* → 2R*n*;
a *reset* mapping *R* : *Θ* × 2R*n* → 2R*n*.

Transition semantics of a hybrid automaton

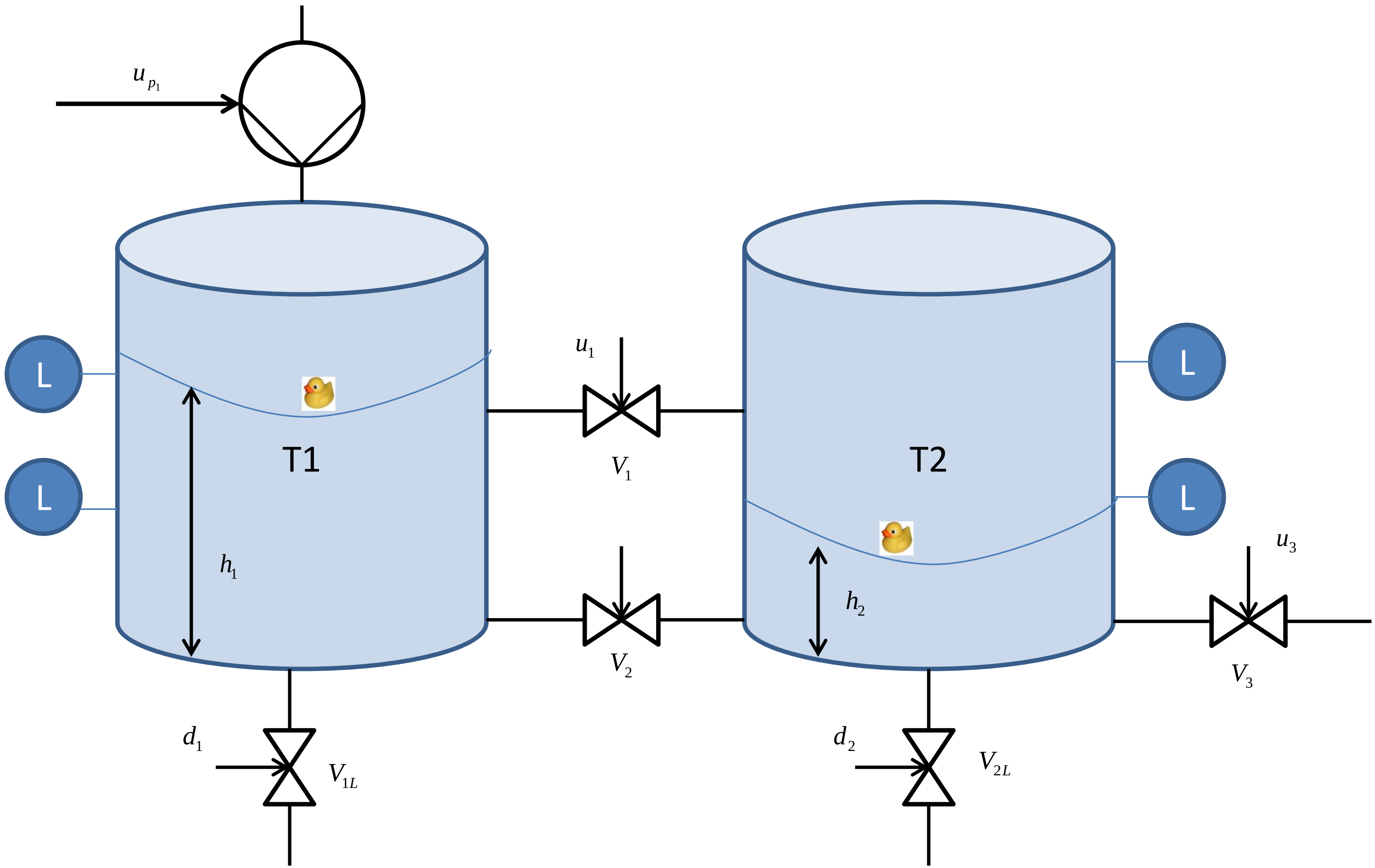


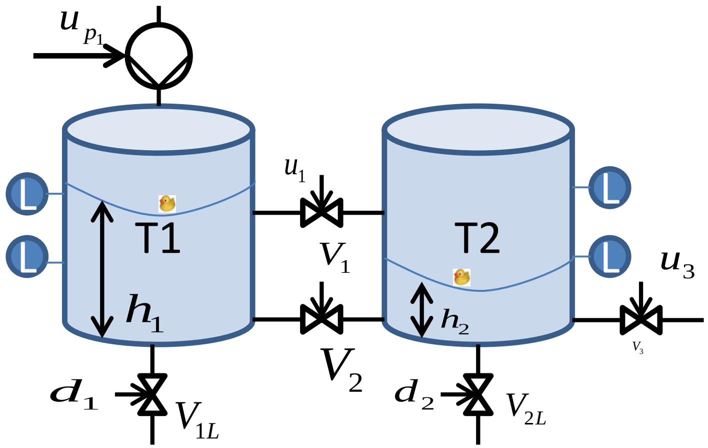
Example: Thermostat



Write the formal definition of this hybrid control system?

Example Two-tank system





The two-tank system has two continuous state variables

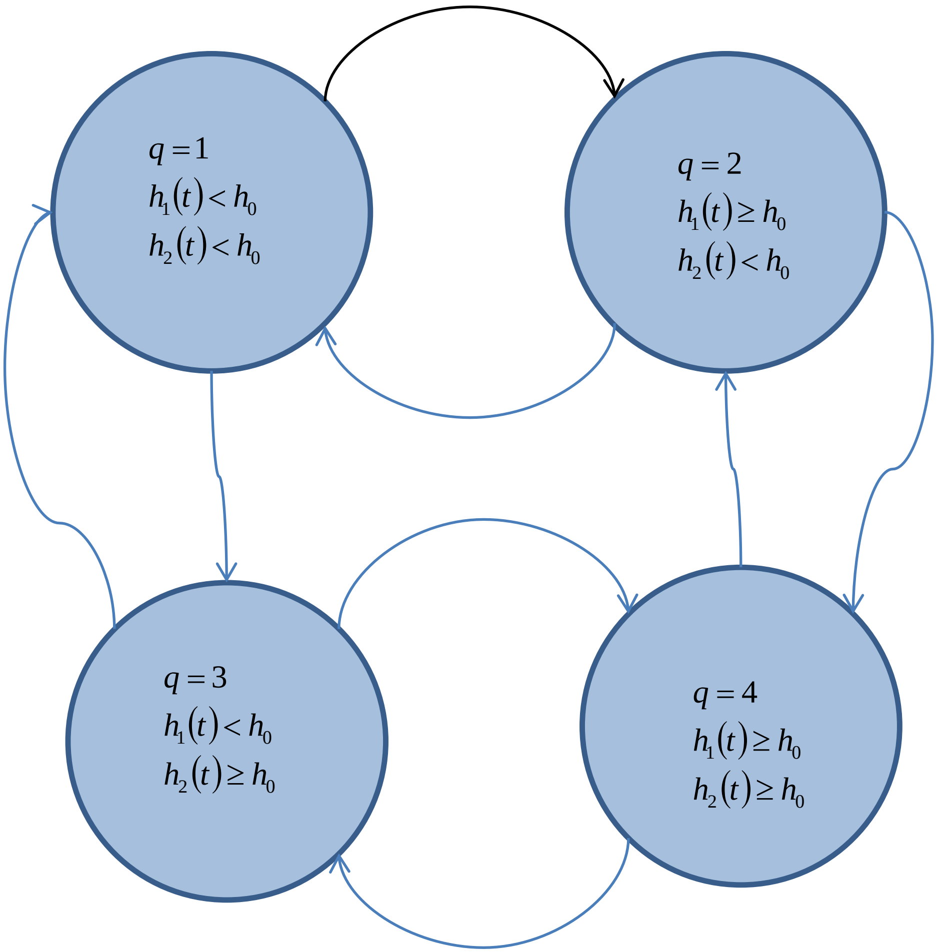
$$x(t) = \begin{pmatrix} h_1(t) & h_2(t) \end{pmatrix}^T, h_i \in R$$

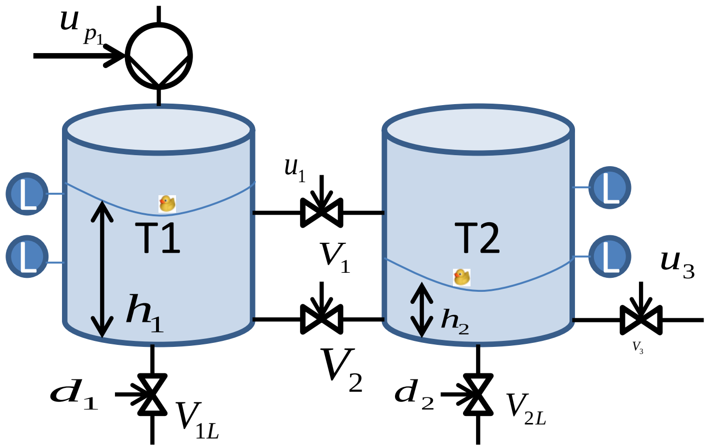
And four discrete states

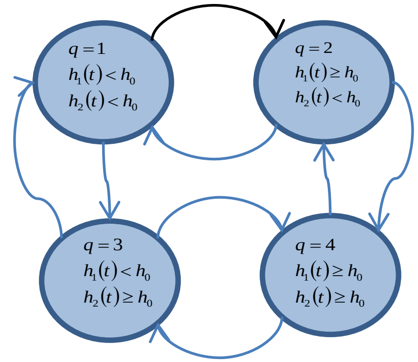
$$q(t) \in \{1,2,3,4\}$$

Discrete modes in dependence of the continuous states:

q(t)	h_1(t)	h_2(t)
1	$<h_0$	$<h_0$
2	$\geq h_0$	$<h_0$
3	$<h_0$	$\geq h_0$
4	$\geq h_0$	$\geq h_0$







$$Q_{ij}^{Vl}(t)=c\cdot\operatorname{sgn}\left(h_i(t)-h_j(t)\right)\cdot\sqrt{2g\cdot|h_i(t)-h_j(t)|}\cdot u_l(t)$$

The nonlinear dynamics follows from Torricelli's law:

Where Q is the water flow from tank T_i into tank T_j through the pipe with valve V_l. c is the flow constant of the valves, $u_l(t)$ is the position of the valve V_l (0 –closed, 1 - open) .

The change of the water volume in a tank $$\dot{V}(t)=\dot{h}(t)\cdot A=\sum Q_{in}(t)-\sum Q_{out}(t)$$

$$\dot{h}_1(t)=\frac{u_{p_1}(t)-Q_{12}^{V_1}(t)-Q_{12}^{V_2}(t)-Q_L^{V_{1L}}(t)}{A}$$

$$\dot{h}_2(t)=\frac{Q_{12}^{V_1}(t)-Q_{12}^{V_2}(t)-Q_L^{V_{2L}}(t)-Q_N^{V_{21L}}(t)}{A}$$

The flow Q depends on the mode q in a following way

$$Q_{12}^{V_1}(t)=\begin{cases}0, & q(t)=1,\\ c\cdot\operatorname{sgn}\left(h_1(t)-h_0\right)\cdot\sqrt{2g\,|h_1(t)-h_0|}\cdot u_1(t), & q(t)=2,\\ c\cdot\operatorname{sgn}\left(h_0-h_2(t)\right)\cdot\sqrt{2g\,|h_0-h_2(t)|}\cdot u_1(t), & q(t)=3,\\ c\cdot\operatorname{sgn}\left(h_1(t)-h_2(t)\right)\cdot\sqrt{2g\,|h_1(t)-h_2(t)|}\cdot u_1(t), & q(t)=4,\end{cases}$$

$$Q_{12}^{V_2(t)}=c\cdot\operatorname{sgn}\left(h_1(t)-h_2(t)\right)\cdot\sqrt{2g\,|h_1(t)-h_2(t)|}\cdot u_2(t),$$

$$Q_N^{V_3(t)}=c\cdot\sqrt{2g\cdot h_2(t)}\cdot u_3(t),$$

$$Q_L^{V_{iL}}=c\cdot\sqrt{2g\cdot h_i(t)}\cdot d_i(t),\quad o=1,2,$$