Advanced Algorithms and Data Structures

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Homework 7

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Task 1 (Alternative minimum spanning tree algorithms) *12 points*

Below, three different algorithms are given in pseudocode. Each algorithm takes a connected graph $G = (N, E)$ and a weight function $w : E \to [0, \infty)$ as input and returns a set T of edges. For each algorithm, show that T is a minimum spanning tree by providing a loop invariant for the **for all** loop,[1] or prove that T is not a minimum spanning tree by giving a counterexample.

1. MAYBE-MST-A (G, w):
   begin
   Sort the edges by weight into nonincreasing order.
   $T := E$
   for all edges e, taken in nonincreasing order by weight
   if $T - \{e\}$ is a connected graph
   $T := T - \{e\}$
   return T
   end MAYBE-MST-A

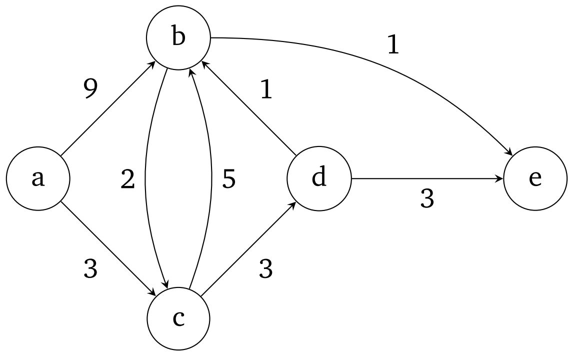
2. MAYBE-MST-B (G, w):
   begin
   $T := \emptyset$
   for all edges e, taken in arbitrary order
   if $T \cup \{e\}$ has no cycles
   $T := T \cup \{e\}$
   return T
   end MAYBE-MST-B

[1]You do not have to prove that your result is really a loop invariant.

3. MAYBE-MST-C (G, w):
   begin
   $T := \emptyset$
   for all edges e, taken in arbitrary order
   $T := T \cup \{e\}$
   if T has a cycle c
   let e' be a maximum-weight edge on c
   $T := T - \{e'\}$
   return T
   end MAYBE-MST-C

Task 2 (Bellman–Ford algorithm) *8 points*

Perform the Bellman–Ford algorithm on the following graph with source node a:



In each relaxation cycle, relax the edges in the following order:

1. (a, b)
2. (b, c)
3. (b, e)
4. (c, b)
5. (c, d)
6. (d, b)
7. (d, e)
8. (a, c)

Draw the situation at the beginning and after each relaxation cycle. To draw a situation, draw the graph and do the following:

- Write the current estimates for the shortest-path weights into the nodes.
- Mark the edges (u, v) for which u is currently a predecessor of v.