Hybrid Systems, Lecture 4: Automata

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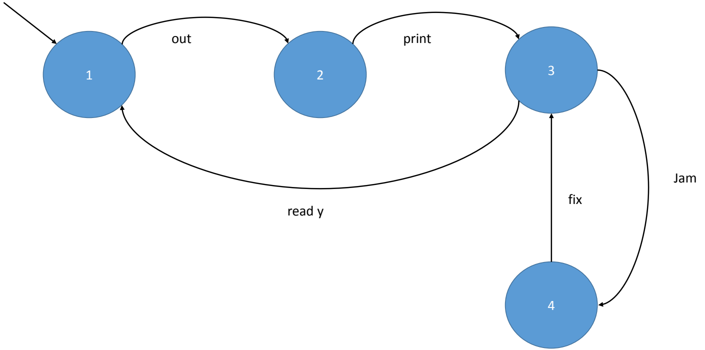
Printer



Transition systems

- Q - is the set of states. What is *state*?
- A - is the set of labels.
- $\rightarrow \subset Q \times A \times Q$ - is the set of transitions.
- Y - is the set of observations (outputs).
- $\langle \cdot \rangle : Q \rightarrow Y$ - observation (output) map.
- Q_0 - is the set of initial states.

Definition

The elements of tuple $S = (Q, A, \rightarrow, Y, \langle \cdot \rangle, Q_0)$ define *transition* system S

If Q and A are finite, the transition system is called *finite*

Transition systems

- A transition between states $q, q' \in Q$ with label $a \in A$ is denoted $q \xrightarrow{a} q'$
- If for some pair (q, a), $Q \in Q$ and $a \in A$ there exist more than one q' then the system is called *nondeterministic*

$$
\begin{aligned}
Q &= \{1, 2, 3, 4\} \\
Q_0 &= 1 \\
A &= \{out, print, ready, jap, fix\} \\
Y &= \{normal, \quad error\} \\
\langle 1 \rangle &= \langle 2 \rangle = \langle 3 \rangle = normal \\
\langle 4 \rangle &= error
\end{aligned}
$$

Execution of the transition systems

- Let $S = (Q, A, \to, Y, \langle\cdot\rangle, Q_0)$ be a transition system.
- The sequence $q_0, a_0, q_1, a_1, \ldots q_{N+1}$ such that $q_i \in Q, \quad i \in [1, N+1]$, $a_i \in A, \quad i \in [1, N]$ is called an *execution* of the system S if $q_0 inQ$ and $q_i \xrightarrow{a_i} q'_{i+1}, i \in [1, N]$
- For the given execution $q_0, a_0, q_1, a_1, \ldots q_{N+1}$ corresponding *external trajectory* is given by $\langle q_0 \rangle, a_0, \langle q_1 \rangle, a_1, \ldots \langle q_{N+1} \rangle$.
- The collection of all possible external trajectories of the transition system is called the *language* of the system.

Automaton

- Q - is the set of states. What is *state*?
- A - is the set of labels.
- $\rightarrow \subset Q \times A \times Q$ - is the set of transitions.
- Y - is the set of observations (outputs).
- $\langle \cdot \rangle : Q \rightarrow Y$ - observation (output) map.
- Q_0 - is the set of initial states.
- Q_m - is the set of marked states.

Definition

The elements of tuple $U = (Q, A, \rightarrow, Y, \langle \cdot \rangle, Q_0, Q_m)$ define *Automaton* system U

If Q and A are finite, the automaton U is called *finite state automaton*.
Definition of nondeterministic automaton is inherited from transition systems.

Execution of automata

- The notion of the *execution* is the same as for transition system.
- For the given execution $q_0, a_0, q_1, a_1, \dots q_{N+1}$ corresponding *trace* is given by $a_0, a_1, \dots a_N$.
- The set of all possible traces is called *generated language of the automaton* U and denoted as $L(U)$

Regular Languages

- An alphabet A is a finite collection of symbols.
- The *string* s over alphabet A is the is a sequence of symbols of A.
- An empty string is denoted by ϵ
- The *length* of the string s is the number of elements is s, denoted by $\mid s \mid$. The length of the empty string is$\mid \epsilon \mid = 0$.
- A language L of the alphabet A is a collection of finite strings over A.

Regular Languages

- Let L_a and L_b be the alphabets over A. *Concatenation* of the languages $L_aL_b = \{s \mid \exists s_a \in L_a, s_b \in L_b, s = s_a s_b\}$
- Let L be the language over the alphabet A. The prefix closure of L is defined as follows

$$\bar{L} = \{s \mid \exists s' : ss' \in L\}.$$

- Let L be the language over the alphabet A. *Kleene closure* of L is defined as follows:

$$L^* = \{\epsilon\} \cup L \cup LL \cup \dots$$

Properties

- The state q is said to be accessible for the automaton $U = (Q, A, \to, Y, \langle\cdot\rangle, Q_0, Q_m)$ if there exists an execution leading to this state.
- Let the automaton $U = (Q, A, \to, Y, \langle\cdot\rangle, Q_0, Q_m)$ the automaton is *blocking* if

$$L_m\overline{(}U) \neq L_m(U)$$