

Annotate where needed and prove partial correctness of  $\{P\} S_1 \parallel S_2 \{Q\}$ :

$$\begin{aligned} \{P\} &\equiv \{x = 5 \wedge y = 7 \wedge v = 0\} \\ \{P_1\} & \\ S_1 : & \\ & \langle \langle x \geq 1 \rightarrow y := x + 2y \rangle \rangle \\ & \square \\ & \langle x < 1 \rightarrow y := x \rangle ; \{P_{11}\} \\ & \langle E! y + 5 \rangle ; x := y - 1 ; \{P_{12}\} \langle C? y \rangle ; z := x + y \\ \{Q_1\} & \\ \parallel & \\ \{P_2\} & \\ S_2 : & \langle E? u \rangle ; v := u - 3 ; \{P_{21}\} \langle C! v + 3 \rangle \\ \{Q_2\} & \\ \{Q\} &\equiv \{z > 9\} \end{aligned}$$

$$P \equiv x = 5 \wedge y = 7 \wedge v = 0$$

$$P_1 \equiv x = 5 \wedge y = 7$$

$$P_2 \equiv v = 0$$

$$Q_1 \equiv z > 9$$

$$Q_2 \equiv u > 10$$

$$P_{11} \equiv y = 19$$

$$P_{12} \equiv y = 19 \wedge x = 18$$

$$P_{21} \equiv v = 21$$

$$Q \equiv z > 9$$

From cooperation tests

$$\text{Coop}(E) : \{P_{11} \wedge P_2\} u = y + 5 \{P_{12}[y - 1/x] \wedge P_{21}[u - 3/v]\}$$

$$\text{Coop}(C) : \{P_{12} \wedge P_{21}\} y = v + 3 \{Q_1[x+y/z] \wedge Q_2\}$$

we extract local axioms for  $S_1$  and  $S_2$  about correct communication actions

$$A_1 = \{ \{P_{11}\} \langle E! y + 5 \rangle \{P_{12}[y - 1/x]\}, \{P_{12}\} \langle C? y \rangle \{Q_1[x+y/z]\} \}$$

$$A_2 = \{ \{P_2\} \langle E? u \rangle \{P_{21}[u - 3/v]\}, \{P_{21}\} \langle C! v + 3 \rangle \{Q_2\} \}$$

For entire parallel program we use the rule

### DML parallel composition

$$\frac{A_1 \vdash \{P_1\} S_1 \{Q_1\} \quad A_2 \vdash \{P_2\} S_2 \{Q_2\} \quad P \Rightarrow P_1 \wedge P_2 \quad Q_1 \wedge Q_2 \Rightarrow Q \quad \text{Coop}(A_1 A_2)}{\vdash \{P\} [\{P_1\} S_1 \{Q_1\} \parallel \{P_2\} S_2 \{Q_2\}] \{Q\}}$$

And for proving  $A_1 \vdash \{P_1\} S_1 \{Q_1\}$  we use the rule

### DML non-deterministic choice

$$\frac{\forall i=1, l: A_i \vdash \{P\} S_i \{Q\},}{A \vdash \{P\} [\sqcup_{i=1}^l S_i] \{Q\}} \quad A =_{\text{def}} \cup_{i=1}^l A_i$$