Search 2

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Outline

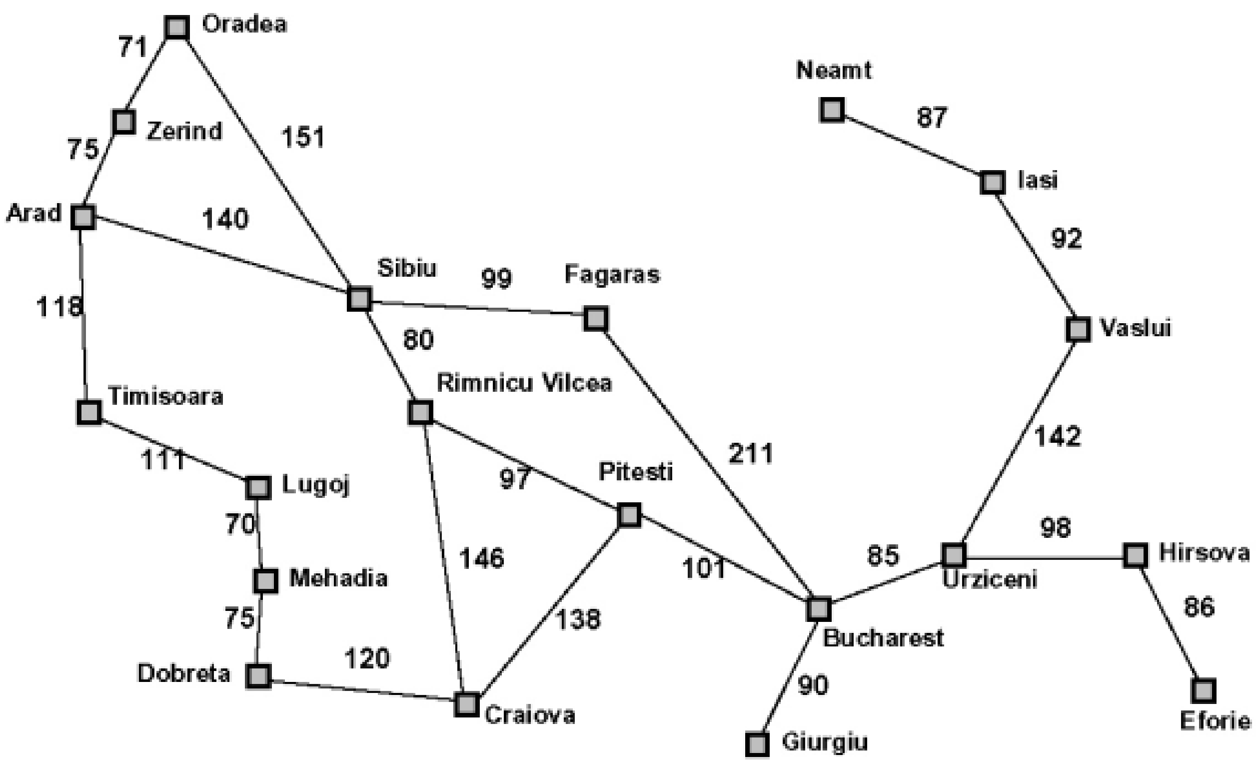
- Informed (Heuristic) search strategies
  - (Greedy) Best-first search
  - A* search
- (Admissible) Heuristic Functions
  - Relaxed problem
  - Subproblem
- Local search algorithms
  - Hill-climbing search
  - Simulated anneal search
  - Local beam search
  - Genetic algorithms
- Online search *
  - Online local search
  - learning in online search

Informed search strategies

- Informed search
  - uses problem-specific knowledge beyond the problem definition
  - finds solution more efficiently than the uninformed search
- Best-first search
  - uses an *evaluation function* $f(n)$ for each node
    - e.g., Measures distance to the goal – lowest evaluation
  - Implementation:
    - Fringe is a queue sorted in increasing order of f-values.
  - Can we really expand the best node first?
    - No! only the one that appears to be best based on $f(n)$.
  - *heuristic function* $h(n)$
    - estimated cost of the cheapest path from node n to a goal node
  - Specific algorithms
    - greedy best-first search
    - A* search

Greedy best-first search

- expand the node that is closest to the goal
- $f(n) = h_{SLD}(n)$ *Straight line distance* heuristic



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
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Timisoara	329
Urziceni	80
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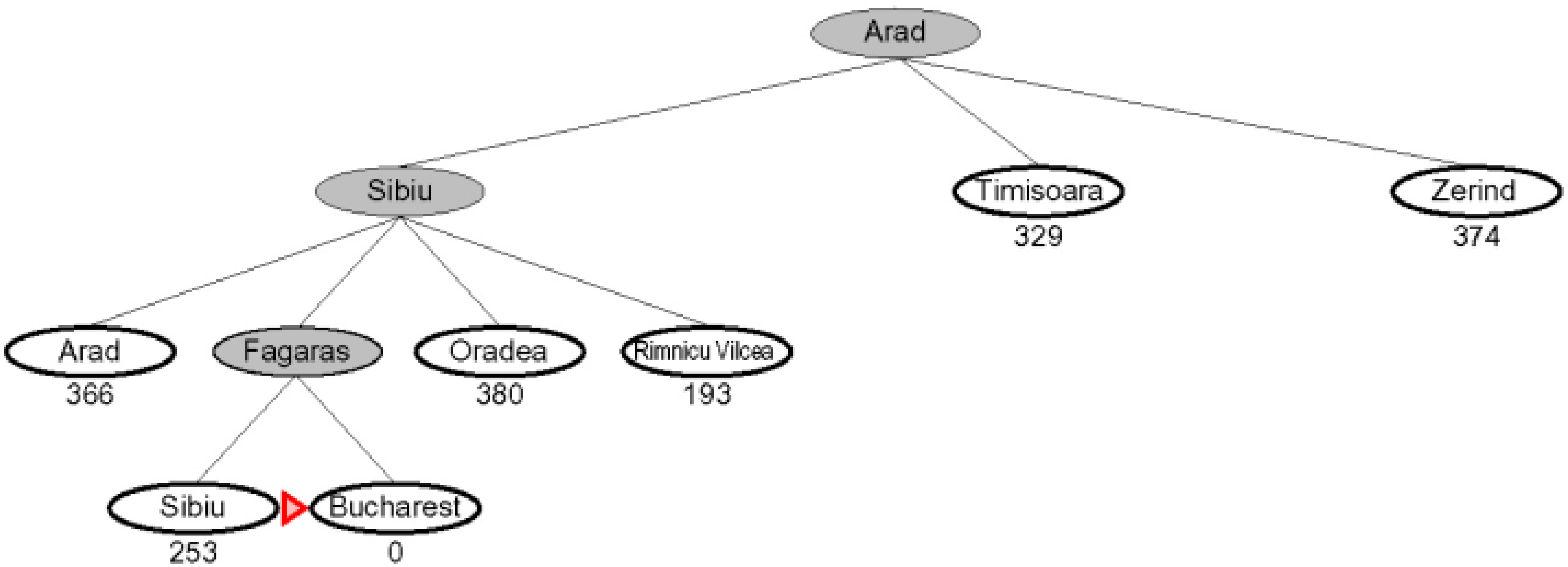
Zerind	374

Greedy best-first search example



Properties of Greedy best-first search

- Complete?

  No – can get stuck in loops, e.g., **Iasi** –> **Neamt** –> **Iasi** –> **Neamt**

  Yes – complete in finite states with repeated-state checking

- Optimal?

  No

- Time?

  $O(b^m)$, but a good heuristic function can give dramatic improvement

- Space?

  $O(b^m)$ – keeps all nodes in memory

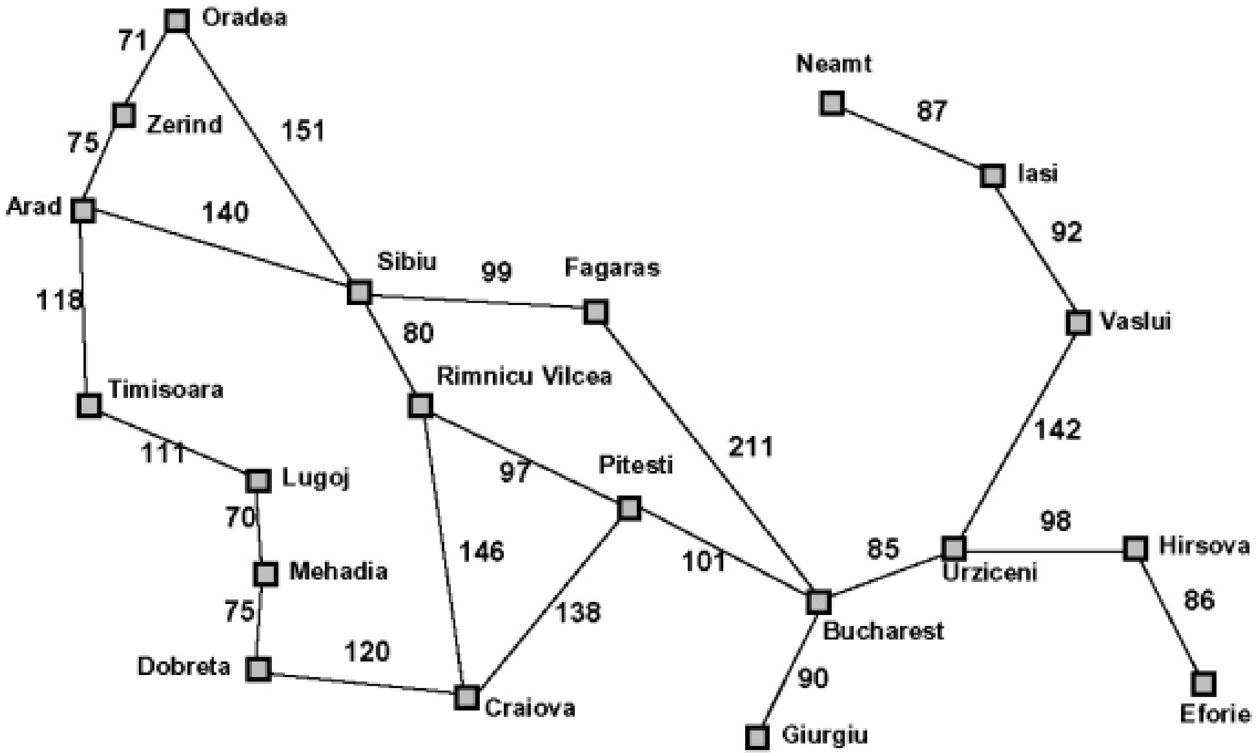
A* search

- evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost to reach the node
  - $h(n)$ = estimated cost to the goal from n
  - $f(n)$ = estimated total cost of path through n to the goal

- an admissible (optimistic) heuristic
  - never overestimates the cost to reach the goal
  - estimates the cost of solving the problem is less than it actually is
  - e.g., $h_{SLD}(n)$ never overestimates the actual road distances

- A* using Tree-Search is optimal if $h(n)$ is admissible
- could get suboptimal solutions using Graph-Search
  - might discard the optimal path to a repeated state if it is not the first one generated
  - a simple solution is to discard the more expensive of any two paths found to the same node (extra memory)

$h_{SLD}(n)$: *Straight line distance* heuristic



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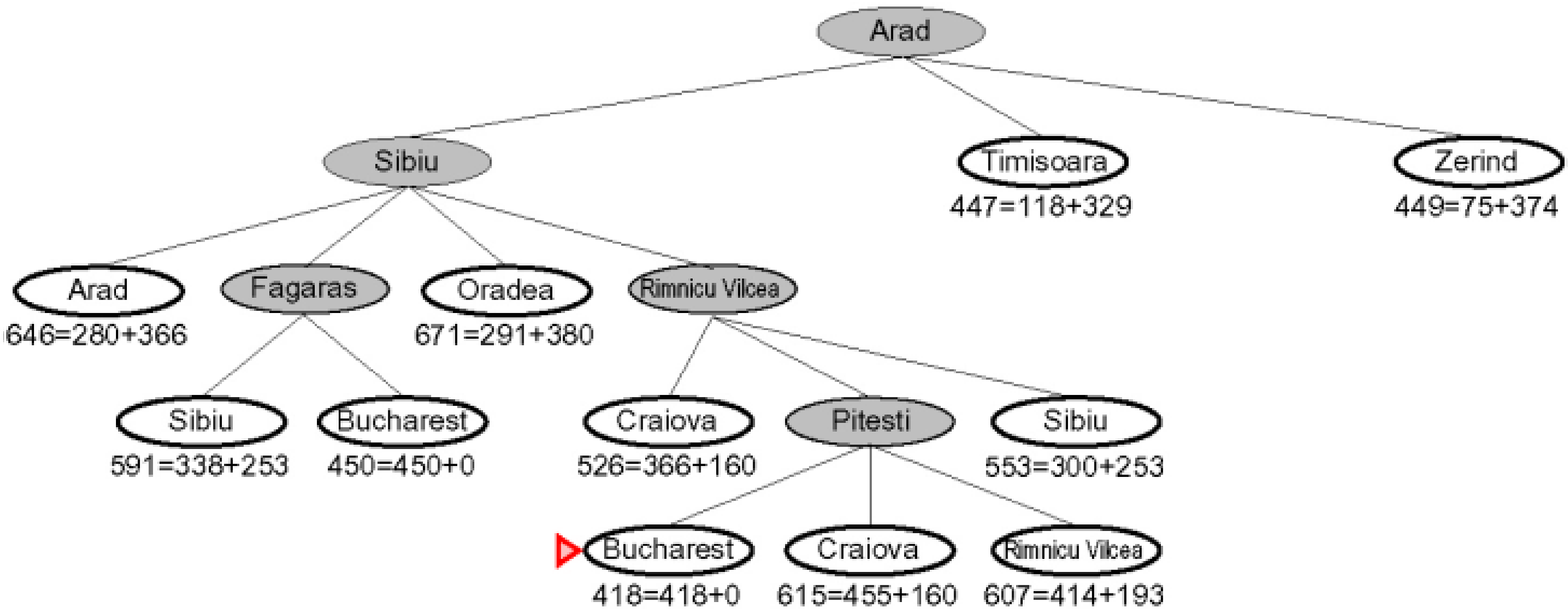
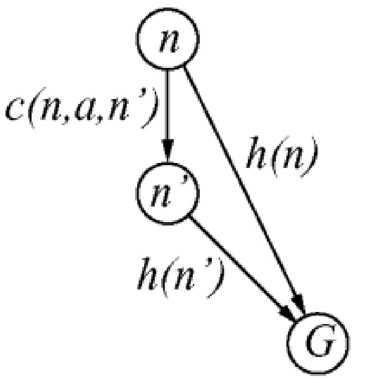
A* search example



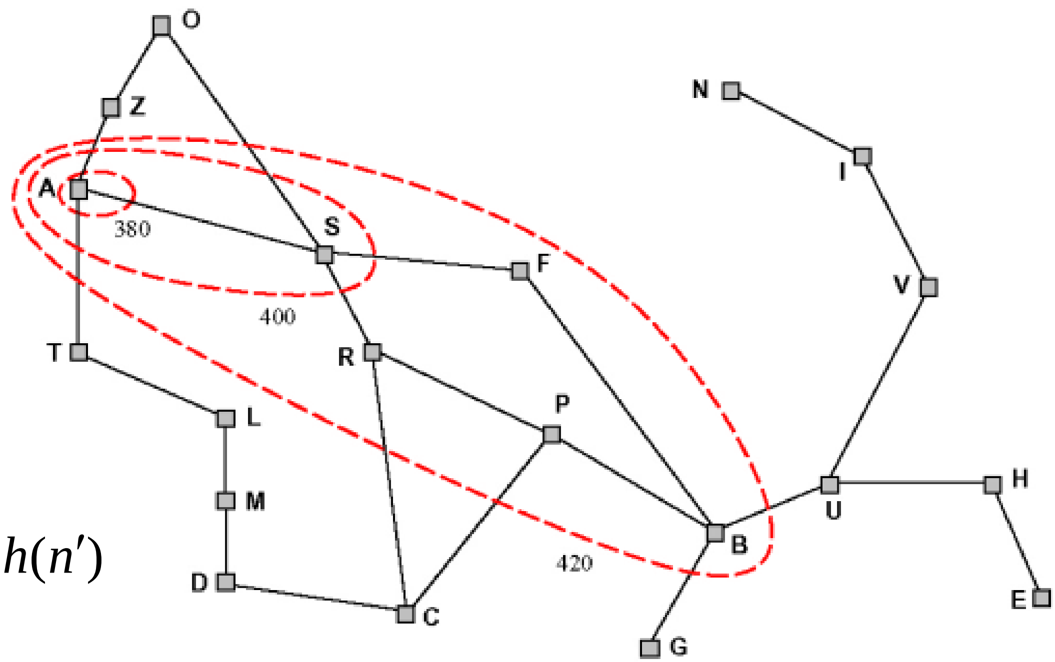
Optimality of A*

- Consistency (monotonicity) $h(n) \le c(n,a,n') + h(n')$
  - *n'* is any successor of *n,* general triangle inequality (*n*, *n'*, and the goal)
  - consistent heuristic is also admissible

- A* using Graph-Search is optimal if *h*(*n*) is consistent
  - the values of *f*(*n*) along any path are nondecreasing



$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\ge g(n) + h(n) = f(n) \end{aligned}$$



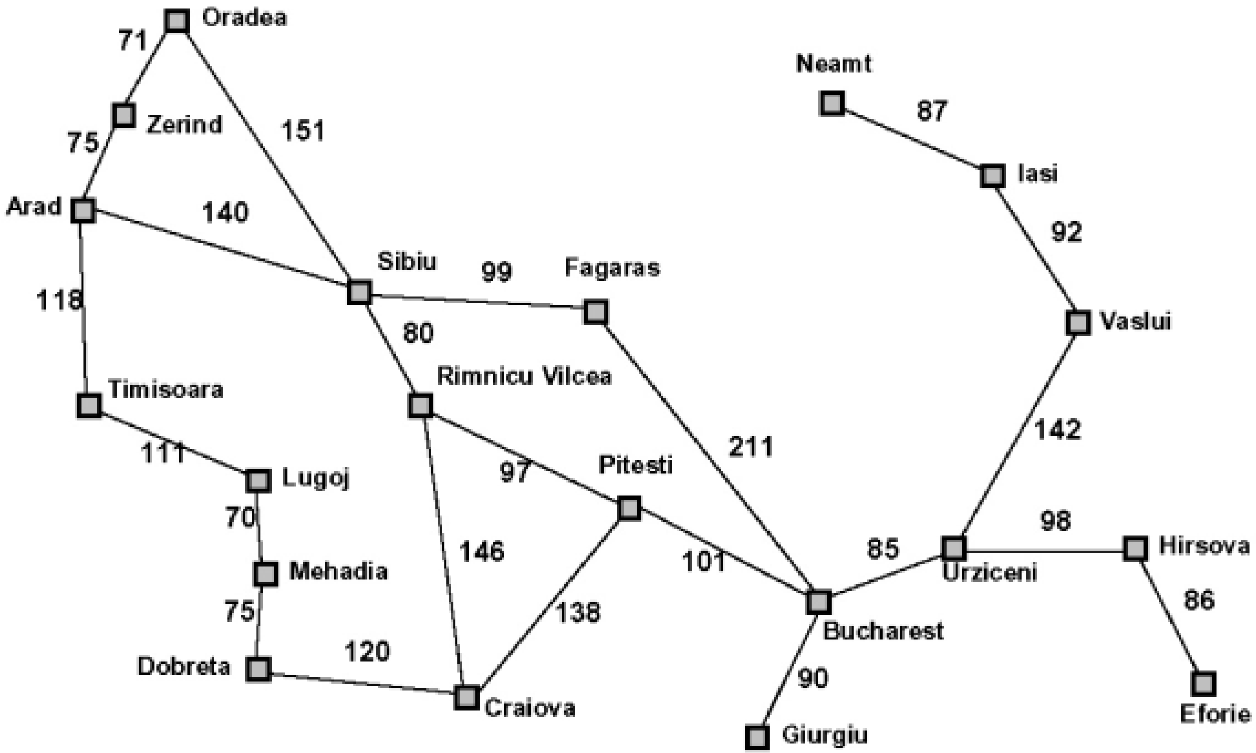
Properties of A*

- Suppose C^* is the cost of the optimal solution path
  - A* expands all nodes with $f(n) < C^*$
  - A* might expand some of nodes with $f(n) = C^*$ on the "goal contour"
  - A* will expand no nodes with $f(n) > C^*$, which are pruned!
  - Pruning: eliminating possibilities from consideration without examination
- A* is optimally efficient for any given heuristic function
  - no other optimal algorithm is guaranteed to expand fewer nodes than A*
  - an algorithm might miss the optimal solution if it does not expand all nodes with $f(n) < C^*$
- A* is complete
- Time complexity
  - exponential number of nodes within the goal contour
- Space complexity
  - keeps all generated nodes in memory
  - runs out of space long before runs out of time

Memory-bounded heuristic search

- Iterative-deepening A* (IDA*)
  - uses *f*-value ($g + h$) as the cutoff
- Recursive best-first search (RBFS)
  - replaces the *f*-value of each node along the path with the best *f*-value of its children
  - remembers the *f*-value of the best leaf in the "forgotten" subtree so that it can reexpand it later if necessary
  - is efficient than IDA* but generates excessive nodes
  - changes mind: go back to pick up the second-best path due to the extension (*f*-value increased) of current best path
  - optimal if $h(n)$ is admissible
  - space complexity is $O(bd)$
  - time complexity depends on the accuracy of $h(n)$ and how often the current best path is changed
- Exponential time complexity of Both IDA* and RBFS
  - cannot check repeated states other than those on the current path when search on Graphs – Should have used more memory (to store the nodes visited)!

$h_{SLD}(n)$: *Straight line distance* heuristic



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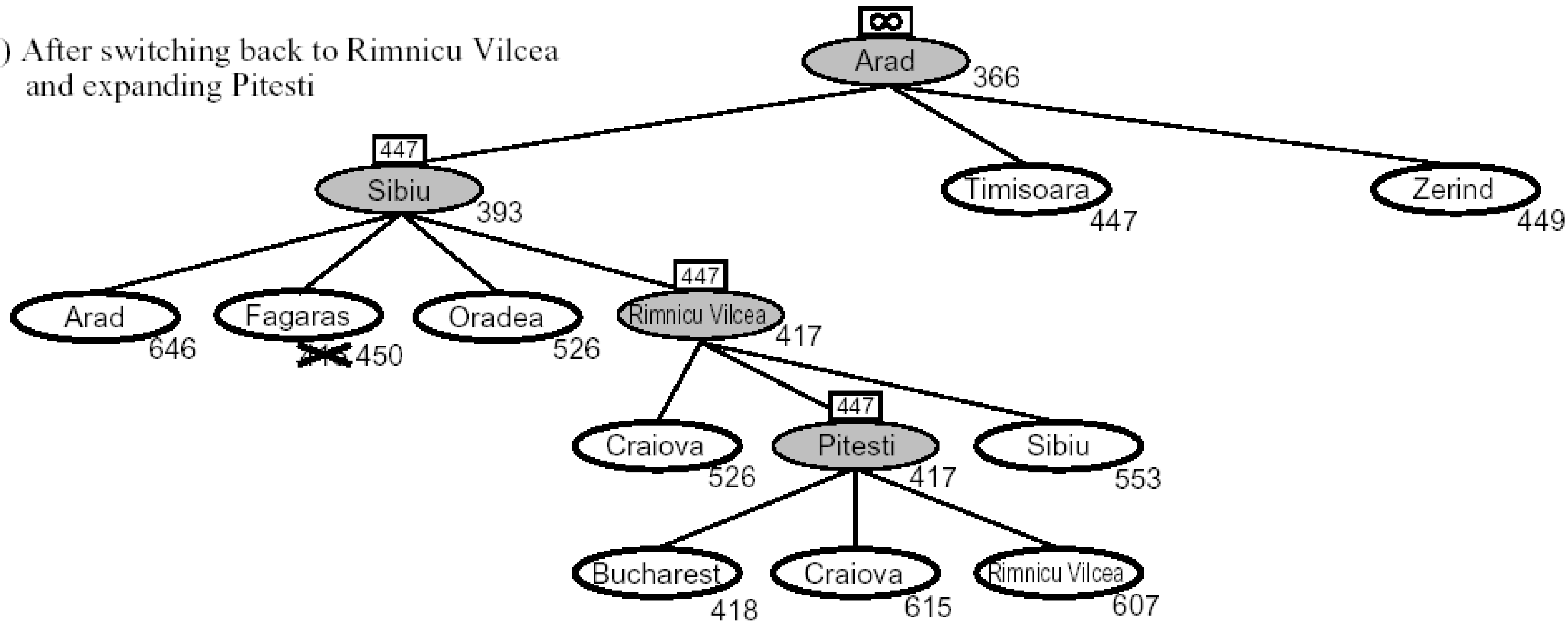
RBFS example

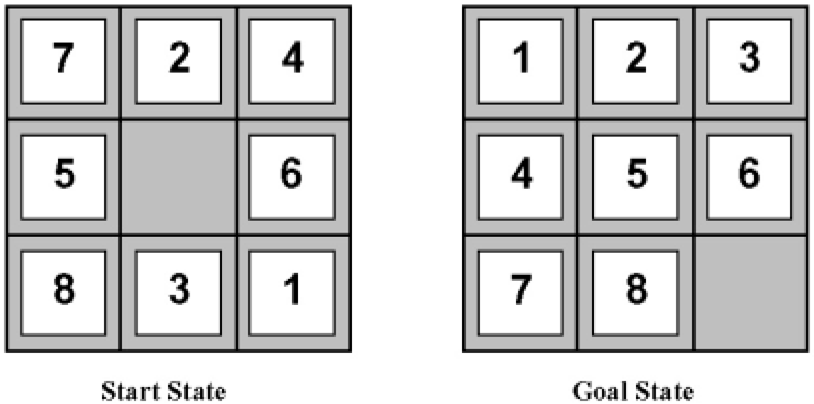
(c) After switching back to Rimnicu Vilcea
and expanding Pitesti



Memory-bounded heuristic search (cont'd)

- SMA* – Simplified MA* (Memory-bounded A*)
  - expands the best leaf node until memory is full
  - then drops the worst leaf node – the one has the highest *f*-value
  - regenerates the subtree only when all other paths have been shown to look worse than the path it has forgotten
  - complete and optimal if there is a solution reachable
  - might be the best general-purpose algorithm for finding optimal solutions
- If there is no way to balance the trade off between time an memory, drop the optimality requirement!

(Admissible) Heuristic Functions



$h_1(n)$ = the number of misplaced tiles

$h_2(n)$ = total **Manhattan** (city block) distance

h_1? = 7 tiles are out of position

h_2? = 4+0+3+3+1+0+2+1 = 14

Effect of heuristic accuracy

- Effective branching factor b^*
  - total # of nodes generated by A* is N, the solution depth is d
  - b^* is b that a uniform tree of depth d containing N+1 nodes would have

$$N + 1 = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$$

  - well-designed heuristic would have a value close to 1
  - h_2 is better than h_1 based on the b^*

- Domination
  - h_2 dominates h_1 if $h_2(n) \geq h_1(n)$ for any node n
  - A* using h_2 will never expand more nodes than A* using h_1

    every node n with $f(n) < C^*$ will be expanded

$$f(n) = g(n) + h(n) < C^* \quad \Rightarrow \quad h(n) < C^* - g(n)$$

$$\Rightarrow \quad h_1(n) \leq h_2(n) < C^* - g(n)$$

  - the larger the better, as long as it does not overestimate!

Inventing admissible heuristic functions

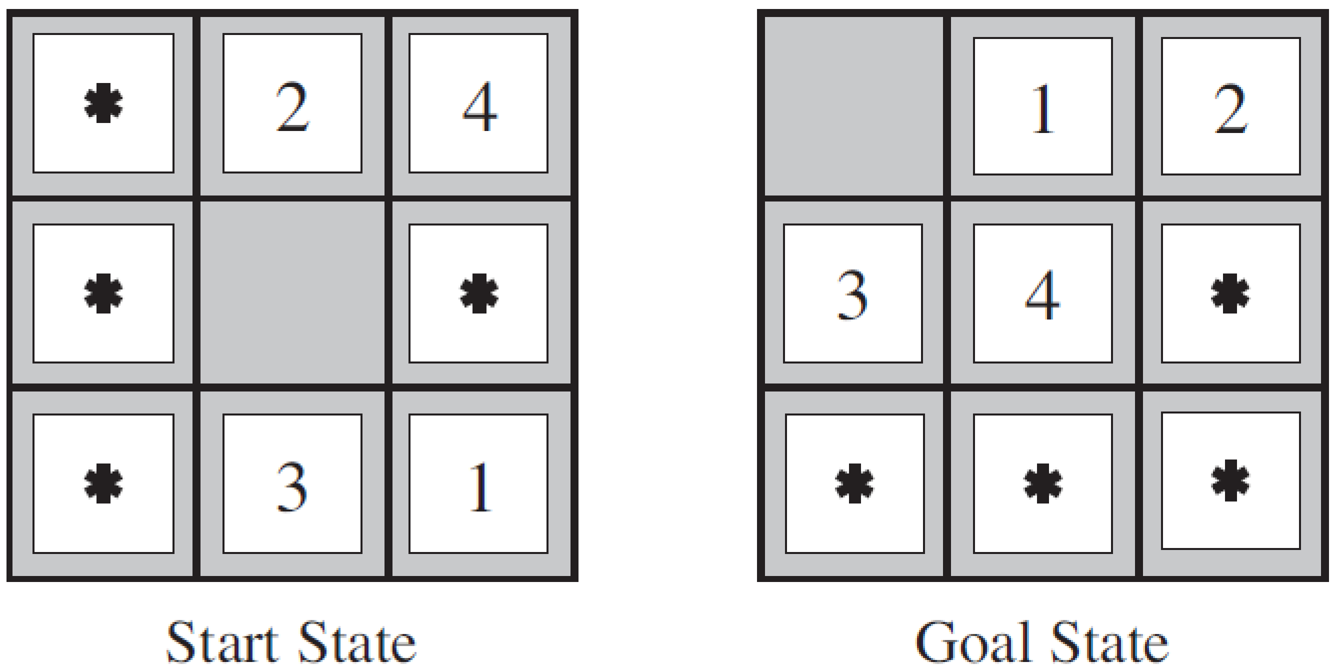
- h_1 and h_2 are solutions to relaxed (simplified) version of the puzzle.
  - If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then h_1 gives the shortest solution
  - If the rules are relaxed so that a tile can move to any adjacent square, then h_2 gives the shortest solution
- Relaxed problem: A problem with fewer restrictions on the actions
  - Admissible heuristics for the original problem can be derived from the optimal (exact) solution to a relaxed problem
  - Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the original problem
  - Which should we choose if none of the h_1 … h_m dominates any of the others?

    We can have the best of all worlds, i.e., use whichever function is most accurate on the current node

$$h(n) = \max\{h_1(n), \ldots, h_m(n)\}$$

- Subproblem *
  - Admissible heuristics for the original problem can also be derived from the solution cost of the subproblem.
- Learning from experience *

Example of subproblems in 8-puzzle



- Acknowledgements

- This set of slides contains several prepared by Hwee Tou Ng and Stuart Russell, available from the AIMA pages.