Advanced Algorithms and Data Structures

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Homework 3

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Task 1 (Implementation of the Karatsuba algorithm) *7 points*

The course website contains Ada code for natural number multiplication. Please note the following about this code:

- Natural numbers are represented as arrays of binary digits. The first element in such an array represents the digit with weight 1, the next the digit with weight 2, the next the digit with weight 4, and so on.
- The package *ITI8590.Natural_Number_Multiplication* contains a procedure *Standard*, which implements the multiplication algorithm taught in elementary school.
- The package *ITI8590.Natural_Number_Multiplication* also contains a private procedure *Add_To*. This procedure takes two parameters that represent natural numbers. It adds the second number to the first, thus modifying it. Note that the first parameter can be an array slice, which makes it possible to effectively shift a number to the left before adding it to another number. This feature is used in the implementation of the *Standard* procedure.
- If the *Value* parameter of *Add_To* or the *Product* parameter of *Standard* is not large enough to hold the result, an exception is raised (this is caused by an out-of-bounds array access).
- The *Put* procedure for natural numbers, which is defined in the *IO* subpackage, outputs digits in the usual order: the most significant digit first and the least significant digit last.

Extend the package *ITI8590.Natural_Number_Multiplication* with a procedure *Karatsuba* that implements the Karatsuba algorithm. The interface of this procedure shall be the same as the interface of the *Standard* procedure. Like the *Standard* procedure, the *Karatsuba* procedure shall impose no constraints on the sizes of its parameter arrays.

Task 2 (Solving of recurrence equations) *6 points*

State for each of the following recurrence equations whether it can be solved by the master theorem, and if it can, give the solution:

1. $T_1(n) = 9T_1(n/3) + 4n^2$

2. $T_2(n) = 3T_2(n/2) + \Theta(n)$
3. $T_3(n) = T_3(n/2) + \Theta(1)$
4. $T_4(n) = T_4(n) + \Theta(1)$
5. $T_5(n) = 4T_5(n/2) + o\left(n^2\right)$
6. $T_6(n) = 2T_6(n/2) + n\sqrt{n}$