Task 1

Name ______________________________

Non-deterministic while program

Annotate and prove the correctness of program **S** provided the following specification is given:

$\{x = 0 \wedge n > 0\}\ S\ \{z = n\}$

```
S ≡
y, x, z := n, 0, 0;
*( x = 0 ∧  y > 0 → z := z + 1; x := 1
   □
   x ≠ 0 ∧  y > 0 → y := y - 1; x := 0)
```

Hint: use z+y-x = n as one conjunct of the invariant

<u>Solution</u>

Annotated program:

$P \equiv \{x = 0 \wedge n > 0\}$

```
C  | C1  | y, x, z := n, 0, 0;
   |       P1 ≡ {x = 0 ∧ n > 0 ∧ y = n ∧ z = 0}
   | C2  | * I ≡ {z + y – x = n}
   |     | C21 | ( [x = 0 ∧ y > 0] → z := z + 1; x := 1      ← b21
   |     |           □
   |     | C22 |   [x ≠ 0 ∧ y > 0] → y := y – 1; x := 0)     ← b22
```

$Q \equiv \{z = n\}$

Partial correctness proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash n > 0 \Rightarrow n > 0}\,(A\Rightarrow A)}{\vdash x=0\wedge n>0 \Rightarrow n>0}\,(\wedge\Rightarrow)}{\vdash x=0\wedge n>0\Rightarrow 0=0\wedge n>0\wedge n=n\wedge 0=0}\,(=)}{\vdash P \Rightarrow P_1\,[n/y,\,0/x,\,0/z]}\,(subs)}{\vdash \{P\}\ C1\ \{P_1\}}\,(:=)$$

$$\dfrac{\overset{1}{\dfrac{}{\vdash P_1\Rightarrow I}}\quad \overset{2}{\dfrac{}{\vdash (I\wedge\neg b_G)\Rightarrow Q}}\quad \overset{3}{\dfrac{}{\vdash \{I\wedge b_{21}\}\ C21\ \{I\}}}\quad \overset{4}{\dfrac{}{\vdash \{I\wedge b_{22}\}\ C22\ \{I\}}}}{\vdash \{P_1\}\ C2\ \{Q\}}\,(*,[])$$

$$\dfrac{\vdash \{P\}\ C1\ \{P_1\} \qquad \vdash \{P_1\}\ C2\ \{Q\}}{\vdash \{P\}\ C\ \{Q\}}\,(;)$$

$$\frac{\frac{\frac{}{\vdash x = 0 \wedge n > 0 \wedge y = n \wedge z = 0 \Rightarrow n = n}\ (\Rightarrow\text{true})}{\vdash x = 0 \wedge n > 0 \wedge y = n \wedge z = 0 \Rightarrow 0 + n - 0 = n}}{\vdash x = 0 \wedge n > 0 \wedge y = n \wedge z = 0 \Rightarrow z + y - x = n}\ (\text{subst})$$

1

$$\vdash z + y - x = n \wedge (x \neq 0 \vee y > 0) \wedge (x = 0 \vee y > 0) \Rightarrow z = n$$

$$\vdash z + y - x = n \wedge \neg(x = 0 \wedge y > 0) \wedge \neg(x \neq 0 \wedge y > 0) \Rightarrow z = n$$

$$\vdash z + y - x = n \wedge \neg(b_{21} \vee b_{22}) \Rightarrow z = n \quad ([.\ /b_{21},\ .\ /b_{22}])$$

$$\vdash z + y - x = n \wedge \neg(b_{21} \vee b_{22}) \Rightarrow z = n \quad (\text{subst})$$

2

$$\vdash I \wedge b_{21} \Rightarrow z + y = n$$

$$\vdash I \wedge b_{21} \Rightarrow z + 1 + y - 1 = n$$

$$\vdash I \wedge b_{21} \Rightarrow I\,[1/x,\ z+1/z] \quad [./x,\ ./z]$$

3

$$\vdash I \wedge b_{22} \Rightarrow z + y - 1 = n$$

$$\vdash I \wedge b_{22} \Rightarrow z + y - 1 - 0 = n$$

$$\vdash I \wedge b_{22} \Rightarrow I\,[0/x,\ y-1/y] \quad [./x,\ ./y]$$

4

Task 2

Name ____________________________________

Shared variable parallel programs (interference test)

Specify assertions for interference test of the program specification

$$P_1 \equiv \{x \leq 4 \wedge y = 2\}$$
$$S_1: \langle x \geq 2 \rightarrow y := y - 2\rangle$$
$$Q_1 \equiv \{y \leq x \wedge x \geq 0\}$$
$$\|$$
$$P_2 \equiv \{x \geq 0 \wedge y \geq 0\}$$
$$S_2: \langle x = 4 \wedge y = 1 \rightarrow z := x - 3\rangle$$
$$Q_2 \equiv \{ y + 2 \leq x \}$$

S1 body does not interfere P2:

$$\vdash \{P_1 \wedge x \geq 2 \wedge P_2\}\ y := y - 2\ \{ P_2 \}$$

S1 body does not interfere Q2:

$$\vdash \{P_1 \wedge x \geq 2 \wedge Q_2\}\ y := y - 2\ \{Q_2 \}$$

S2 body does not interfere P1:

$$\vdash \{P_2 \wedge x = 4 \wedge y = 1 \wedge P_1\}\ z := x - 3\ \{P_1 \}$$

S2 body does not interfere Q1:

$$\vdash \{P_2 \wedge x = 4 \wedge y = 1 \wedge Q_1\}\ z := x - 3\ \{Q_1 \}$$

Task 3

Name ______________________________________

Parallel programs with message passing (cooperation test).
Specify assertions for cooperation test.

$P_1 \equiv \{x = 5 \wedge y = 7\}$
$S_1 : \langle E!\ y - 2\rangle;\ \langle x := y - 1\rangle\ \{x = 6 \wedge y = 7\};\ \langle C!\ x{+}5\rangle$
$Q_1 \equiv \{y < 9 \wedge x > 0\}$
$\|$
$P_2 \equiv \{u = 0\}$
$S_2 : \langle E?\ u\rangle;\ \{u < 10\}\langle C?\ u\rangle$
$Q_2 \equiv \{u > 9\}$

Solution:

Test for channel *E*:
$\vdash \{ P_1 \wedge P_2\}\ u := y - 2\ \{(x = 6 \wedge y = 7)\ [y - 1/\ x] \wedge u < 10\}$

Test for channel *C*:
$\vdash \{x = 6 \wedge y = 7\}\ u := x + 5\ \{ Q_1 \wedge Q_2 \}$