

Data Mining, Lecture 13-2

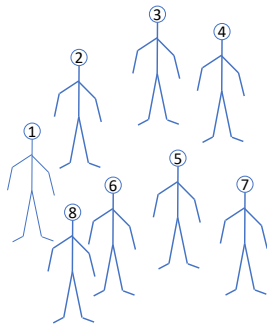
Social Networks Analysis

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Individuals → Graphs

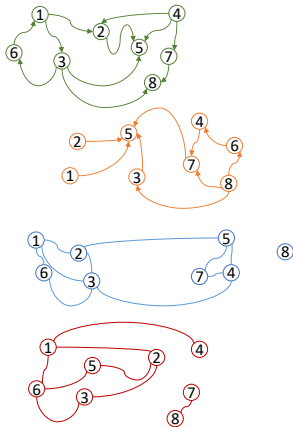


Phone

Bank acc.

Social Env.

Chat & voip



Preliminaries and properties

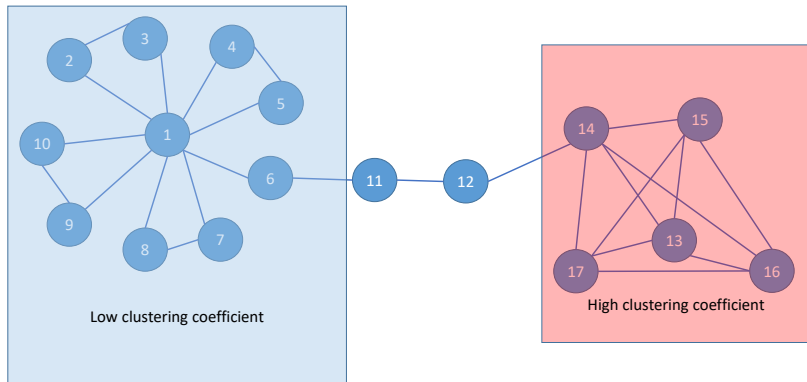
- Let us assume that social networks may be structured as a graph, $G = (N, A)$ where N is the set of nodes and A is the set of edges. Each individual in the networks is represented by a node in N and referred as *actor*. The edges represent connections between the actor.
- Assume that G is undirected.
- In some cases nodes may have content associated with them.
- Usually each node is associated with an actor (human individual).

Key properties

- *Homophily (Assortative mixing)*: nodes that are connected to one another are more likely to have similar properties.
- *Triadic closure*: If two individuals in a social network have a friend in common, then it is more likely that they are either connected or will eventually become connected in the future. **Observe the reference to some dynamics.** Implies an inherent correlation in the edge structure of the network
- *The clustering coefficient of the network* is the measure of the inherent tendency of a network to cluster. Similar to the Hopkins statistic for multidimensional data.
- Let $S_i \subseteq N$ be the subset of nodes connected to the node $i \in N$, let the cardinality of S_i be n_i . The local clustering coefficient is defined as follows:

$$\eta(i) = \frac{|\{j, k\} \in A : j \in S_i, k \in S_i|}{\binom{n_i}{2}}$$

Clustering Coefficient



Associations

- **Associates:** Basic level, do not share any interests.
- **Useful friends:** Information sharing.
- **Fun friends:** Socialise together, no emotional connection.
- **Favor friends:** may help each other, no emotional connection.
- **Help mates:** Combination of two previous.
- **Comforters:** Help mates with emotional connection.
- **Confidants:** Share personal emotional information, socialize together, unable to help each other.
- **Soulmates** : Most probably the tightest type of connection.
- number of stronger associations is always smaller than number of weak associations.

The rule of 5-15-50-150-500

- Internal circle 5
- Sympathy group 12 -15
- More or less regular group 50
- Stable social group 150 (Dunabar's value).
- Weak associations 500

Key properties: Dynamics of Network Formation

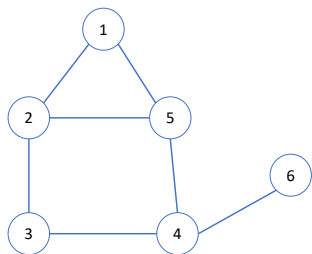
- Preferential attachment: In a growing network, the likelihood of a node receiving new edges increases with its degree. Highly connected individuals will typically find it easier to make new connections.
- Let $\pi(i)$ is the probability a newly added node attaches itself to an existing node i . The model of $\pi(i)$

$$\pi(i) \propto \text{Degree}(i)^\alpha$$

where the value of the parameter α depend on domain where form the network is drawn.

- *Small world property*: Most real networks are assumed to be “small world”. Average path growth is $\log(n(t))$.

Degrees and frequencies



Node	Deg.
1	2
2	3
3	2
4	3
5	3
6	1

Deg.	Frequency
1	1/6
2	2/6
3	3/6

Key properties: Dynamics of Network Formation II

- *Densification*: Almost all real-world networks add more nodes and edges over time than are deleted.

$$e(t) \propto n(t)^\beta$$

where $e(t)$ is the number of edges, exponent of β is the value between 1 and 2.

- *Shrinking diameters*: In most real-world networks, as the network densifies, the average distances between the nodes shrink over time.
- *Giant connected component*: As the network densifies over time, a giant connected component emerges.
- *Power-Law Degree Distributions*: a small minority of high-degree nodes continue to attract most of the newly added nodes:

$$P(k) \propto k^{-\gamma}$$

where γ ranges between 2 and 3; larger values of γ lead to more small degree nodes.

Key properties

- *Degree Centrality and Prestige* The degree centrality $C_D(i)$ of a node i of an undirected network is equal to the degree of the node, divided by the maximum possible degree of the nodes.

$$C_D(i) = \frac{\text{Degree}(i)}{n - 1}.$$

- *Degree prestige* is defined for directed networks only.

$$P_D(i) = \frac{\text{InDegree}(i)}{n - 1}.$$

- *The gregariousness of a node:* (extension of the centrality to outdegree):

$$G_D(i) = \frac{\text{OutDegree}(i)}{n - 1}.$$

The gregariousness of a node defines a different qualitative notion than prestige because it quantifies the propensity of an individual to seek out new connections.

Closeness Centrality and Proximity Prestige

- *Closeness centrality*: for undirected and connected networks. The average shortest path distance, starting from node i , is denoted by $\text{AvDist}(i)$:

$$\text{AvDist}(i) = \frac{\sum_{j=1}^n \text{Dist}(i, j)}{n - 1}.$$

The closeness centrality is the inverse of the average distance of other nodes to node i .

$$C_C(i) = 1/\text{AvDist}(i)$$

ranges between 0 and 1.

Closeness Centrality and Proximity Prestige

- *Proximity prestige*: defined for the directed networks. $\text{Influence}(i)$ corresponds to all recursively defined "followers" of i .

$$\text{AvDist}(i) = \frac{\sum_{j \in \text{Influence}(i)} \text{Dist}(i, j)}{|\text{Influence}(i)|}.$$

- Nodes that have less influence should be penalized.

$$\text{InfluenceFraction}(i) = \frac{\text{Influence}(i)}{n - 1}.$$

- Proximity prestige may be defined as follows:

$$P_P(i) = \frac{\text{InfluenceFraction}(i)}{\text{AvDist}(i)}.$$

Betweenness Centrality

- Closeness centrality does not account the degree of importance (criticality) of the node with respect to the number of shortest paths goes through it.
- Let $q_{j,k}$ denotes the number of shortest paths between nodes j and k . Let $q_{j,k}(i)$ be the the number of shortest paths goes through the node i .
- denote by $f_{jk}(i)$ the fraction of pairs that pass through the node i

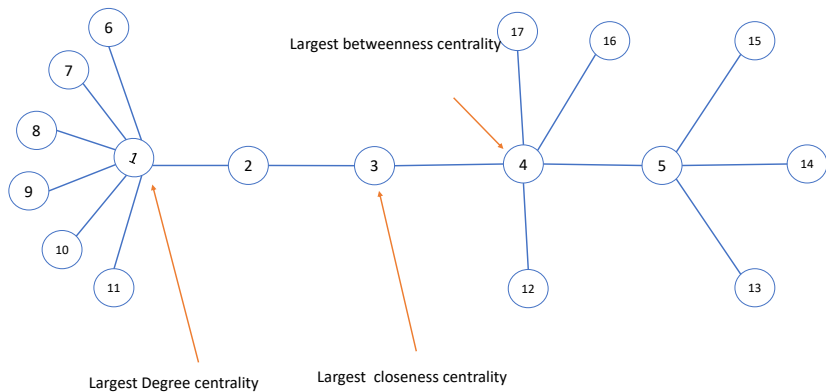
$$f_{jk}(i) = \frac{q_{j,k}(i)}{q_{j,k}}.$$

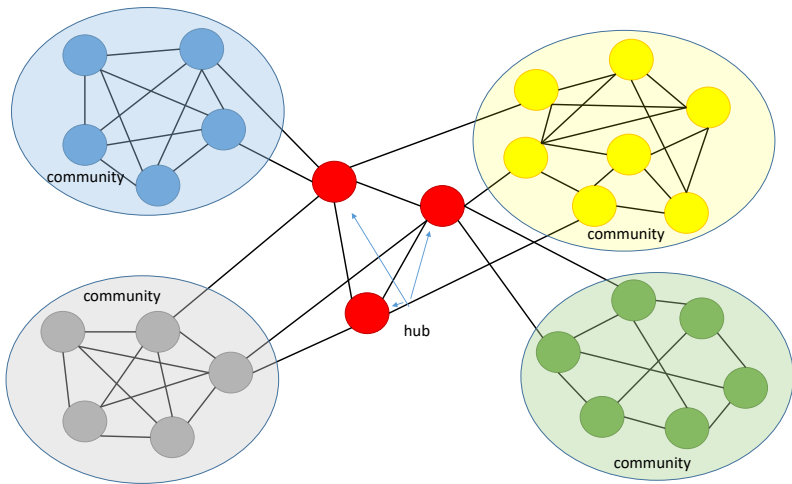
- *The betweenness centrality:* is defined as follows:

$$C_B(i) = \frac{\sum_{j < k} f_{jk}(i)}{\binom{n}{2}}.$$

May be generalized for disconnected networks. May be redesigned for edges.

Centrality and Prestige





Communities

- Giant (gigantic) component - connected component (where all or nearly all the vertexes are connected), which weight in the network is constant.

$$\lim_{N \rightarrow \infty} \frac{N_1}{N} = c > 0.$$

- Community structure. Connected components and weak associations between them.

Community detection

- “Community detection” is an approximate synonym for “clustering” in the context of social network analysis.
- Methods of “graph partitioning” may be applied.
- k-means and other non specific clustering algorithm may not be easily applied here.
- Different parts of the social network have different edge densities. In other words, the local clustering coefficients in distinct parts of the social network are typically quite different.
- Kernighan–Lin Algorithm.
- Girvan–Newman Algorithm.
- Multilevel Graph Partitioning: METIS
- Spectral Clustering

Collective Classification

- Iterative Classification Algorithm.
- Label Propagation with Random Walks.
- Supervised Spectral Methods.

Link Prediction

- Structural measure. Structural measures typically use the principle of triadic closure to make predictions.
- Content-based measures. In these cases, the principle of homophily is used to make predictions.

Neighbourhood based measures

- Common neighbour based measure between nodes i and j .

$$C_N(i, j) = |S_i \cap S_j|.$$

The major weakness of the common-neighbor measure is that it does not account for the relative number of common neighbors between them as compared to the number of other connections.

- Jaccard Measure:

$$J_M(i, j) = \frac{|S_i \cap S_j|}{|S_i \cup S_j|}.$$

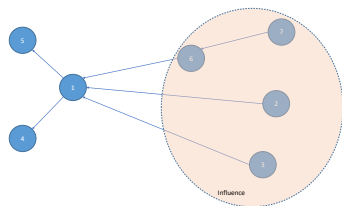
Main drawback is that it does not adjust well to the degrees of their intermediate neighbors.

- Adamic–Adar Measure:

$$A_A(i, j) = \sum_{k \in S_i \cap S_j} \frac{1}{\log(|S_k|)}$$

Influence

- For the oriented graphs one may define prestige in the context of the close neighborhood.
- Influence I_i of the vertex v_i is the subset of the vertexes such that they are terminal for at least one path with origin in the vertex v_i .



Practice

- Install "igraph" package for "R".
- The practice is based on <https://kateto.net/networks-r-igraph> .
- Download the data set from kateto.net/netscix2016 .