Hybrid Systems, Lecture 2: Controls Systems (Reminder)

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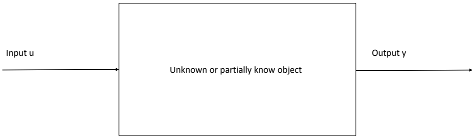
Organisation clarified

No Plagiarism in any of your tests of final project!!!. You should cite all the references, including libraries you use to complete your computational assignments. The student should be able to explain the meaning of all the computations performed, interpret and present the results. Grading:

- Two tests, each gives you max 10 % of the final grade. In the end of the term you will be given one make-up attempt.
- Two home assignments(followed by presentation). Each gives you max 10% of the final grade. If you fail to prepare your presentation no make-up is possible.
- Final project gives you max 60% of the final grade. It will require to design and implement hybrid system. Perform numeric simulation and interpret the results. On the examination day, you will be given three minutes to state the problem, and present the results followed by more detailed analysis of your work. You may be asked to perform simulation or even to introduce some changes into implementation of your system. Usually it takes ten to fifteen days to complete the project.

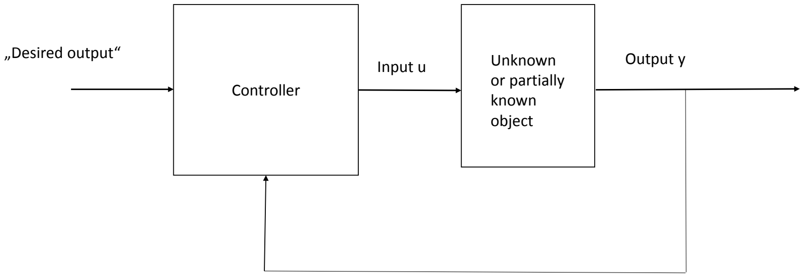
Controlled object



An entity whose changes in time may be externally influenced and measured.

The notion of feedback

According to
`http://www.merriam-webster.com/dictionary/feedback`
: helpful information or criticism that is given to someone to say what can be done to improve a performance, product, etc.
: something (such as information or electricity) that is returned to a machine, system, or process
: an annoying and unwanted sound caused by signals being returned to an electronic sound system


„Desired output“
Controller
Input u
Unknown
or partially
known
object
Output y

Representations

- Input-output description:

$$y^{(n)} = f(y, \dot{y}, \ddot{y}, \ldots, y^{n-1}, u, \dot{u}, \ldots, u^{(m)})$$

- State - space description:

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases}$$

- Transfer function:

$$H(s) = \frac{Y(s)}{X(s)}$$

Continuous *versus* Discrete

- Continuous time system

$$y^{(n)} = f(y, \dot{y}, \ddot{y}, \dots, y^{n-1}, u, \dot{u}, \dots, u^{(m)}).$$

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases}$$

$$H(s) = \frac{Y(s)}{X(s)}$$

- Discrete time system

$$y(t) = f(y(t-1), \dots, y(t-n), u(t-1), \dots, u(t-m)).$$

$$\begin{cases} x(t+1) = f(x(t), u(t)) \\ y(t) = g(x(t), u(t)) \end{cases}$$

$$H(s) = \frac{Y(z)}{X(z)}$$

Input - output description

$$y^{(n)} = f(y, \dot{y}, \ddot{y}, \dots, y^{n-1}, u, \dot{u}, \dots, u^{(m)})$$

$$y(t) = f(y(t-1), \dots, y(t-n), u(t-1), \dots, u(t-m)).$$

- Well suited for parameter identification.
- Not very well suited for control synthesis.

Linear *versus* Nonlinear

- Linear:

$$\begin{cases} \dot{x} = Ax + Bu(t) \\ y = Cx + Du(t) \end{cases}$$

- nonlinear

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases}$$

The concept of state

One of the possible definitions by [Simrock]
The system state x of a system at any time t is the *amount of information* that, together with all inputs for $t \geq t_0$, uniquely determines the behaviour of the system for all $t \geq t_0$.
Sate space representation of the system:

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

Main properties

- Controllability
- Observability
- Stability

Controllability

Controllability is the property of possessing and ability to transform the system from one state to another.

Consider:

$$x(t+1) = Ax(t) + Bu(t) \tag{1}$$

Definition

The linear time invariant control system 1 is said to be completely controllable if for any t_0, any initial state $x(t_0) = x_0$, and any given final state x_f, there exists a final time $t_1 > t_0$, and a control $u : [t_0, t_1] \to \mathbb{R}^l$ such that $x(t_1) = x_f$.

How to check?

Theorem

The linear time-invariant system 1 is completely controllable if and only if the Klaman controllability matrix

$$\mathcal{C}(A, B) = \begin{bmatrix} B & AB & A^2B \dots A^{m-1}B \end{bmatrix} \in \mathbb{R}^{mxml}$$

has rank m

Controllability: Example

$$\begin{cases} \dot{x}_1 &= a_1 x_1 + a_2 x_2 + u(t) \\ \dot{x}_2 &= x_2. \end{cases}$$

Clearly, u has no influence on x_2 and therefore system is not completely controllable. Check rank condition yourself.

Observability

Observability is the property of possessing an ability to reconstruct the state of the system by measuring its output only.

Definition

The linear control system

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

is said to be completely observable if for any $t_0$and any initial state $x(t_0) = x_0$, there exists a finite time $t_1 > t_0$ such that knowledge of $u(\cdot)$ and $y(\cdot)$ for $t \in [t_0, t_1]$ suffices to determine x_0 uniquely

Observability

How to check?

Theorem

The linear time-invariant control system (5) is completely controllable if and only if Kalman observability matrix

$$\mathcal{O} = \mathcal{O}(A, C) = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{m-1} \end{bmatrix} \in \mathbb{R}^{mnxm}$$

has rank m.

Observability:example

Check whether the following system is completely observable or not

$$\begin{cases} \dot{x} = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y = x_1 \end{cases}$$

Stability

There is no single concept of stability, and many different definitions are possible. We will review this subject in a more detailed way, while talking about stability of hybrid system The following theorem provides condition to verify if given linear system is stable or not.

Theorem

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

The system 5 is asymptotically stable if and only if every eigenvalue of A has a negative real part.

Case of nonlinear system

Things are getting more complicated

- the notions remain the same but to check properties is more complicated
- new notion is "linearizability"
- ...